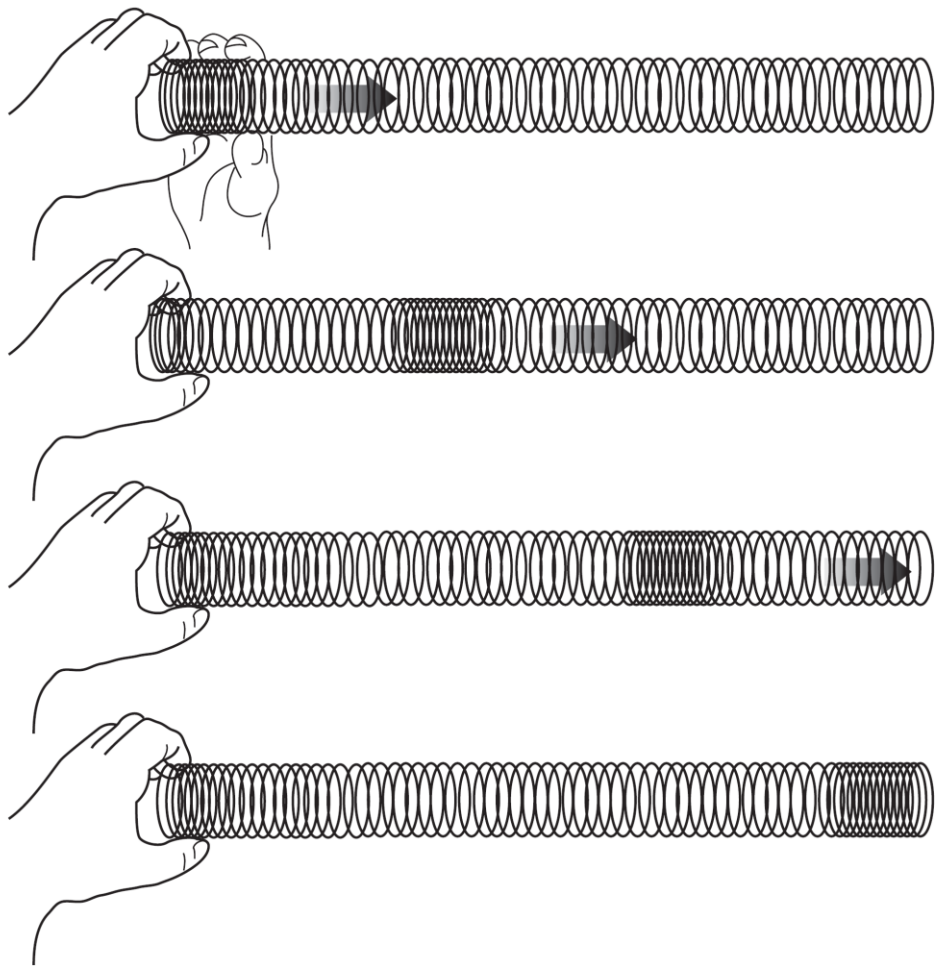
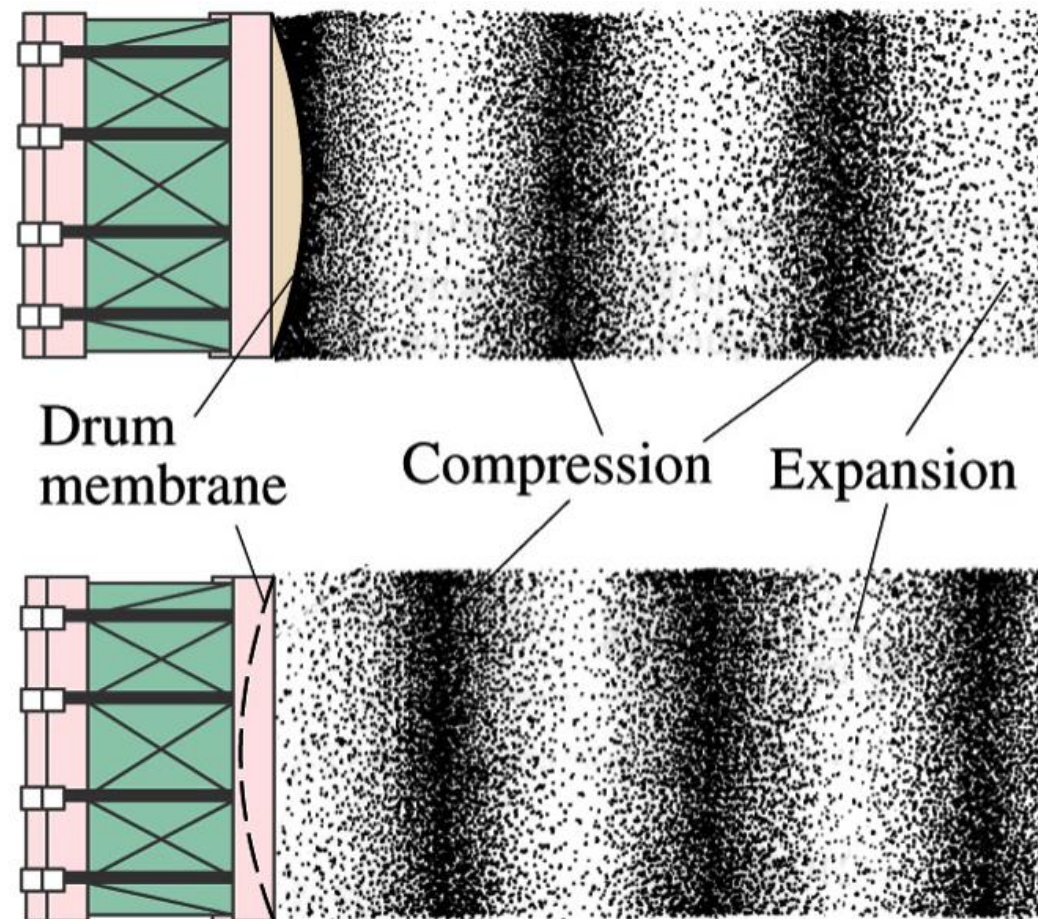


Week 13 and 14:
Waves

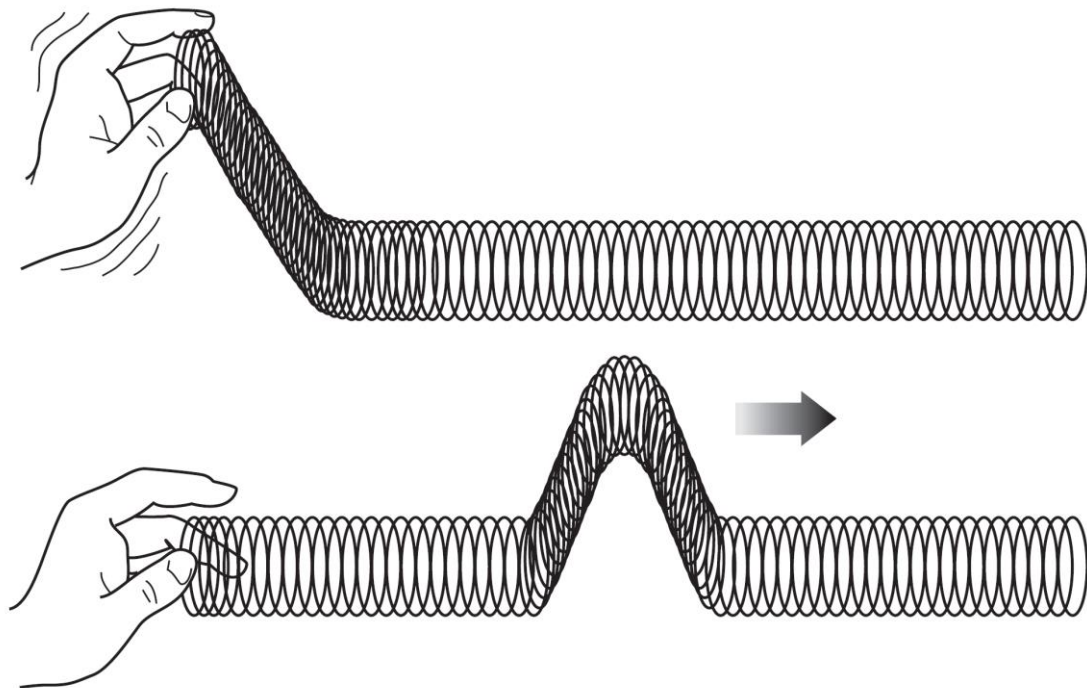
Longitudinal wave in a spring



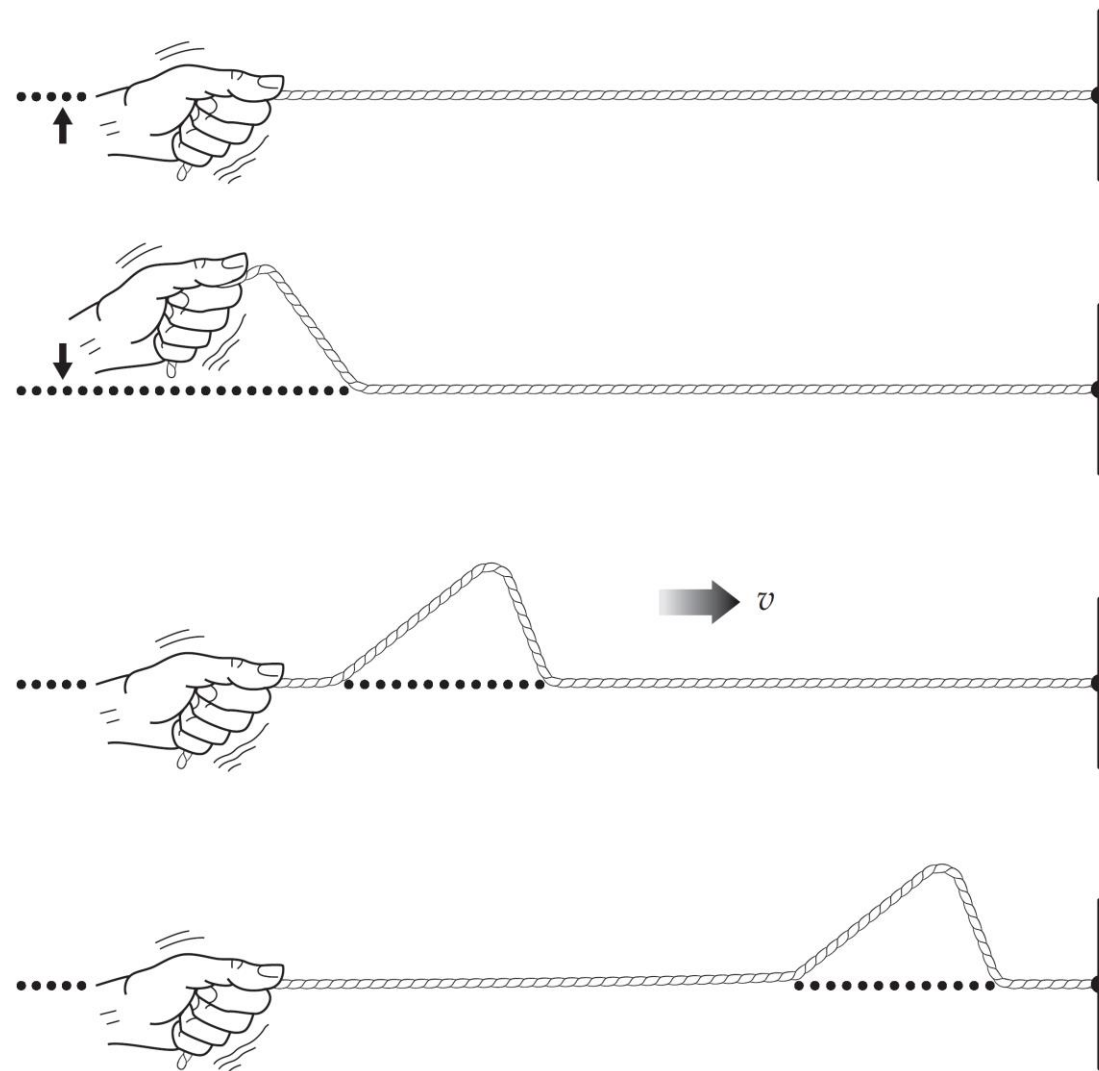
Longitudinal wave in a gas



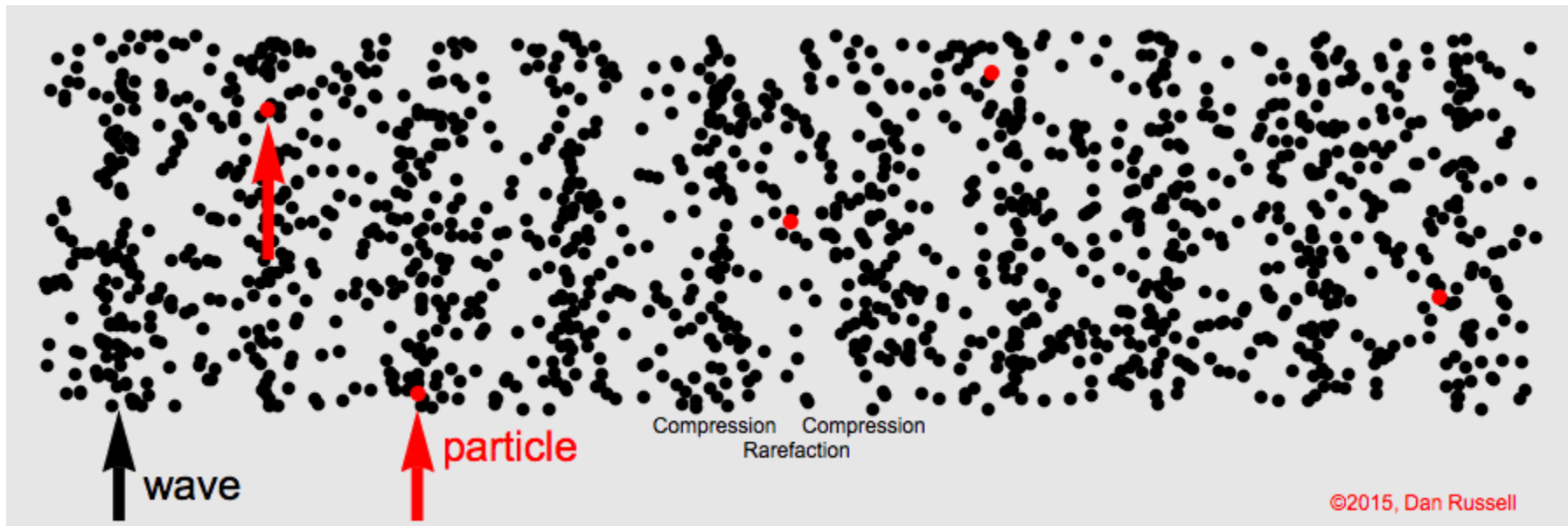
transverse wave in a spring



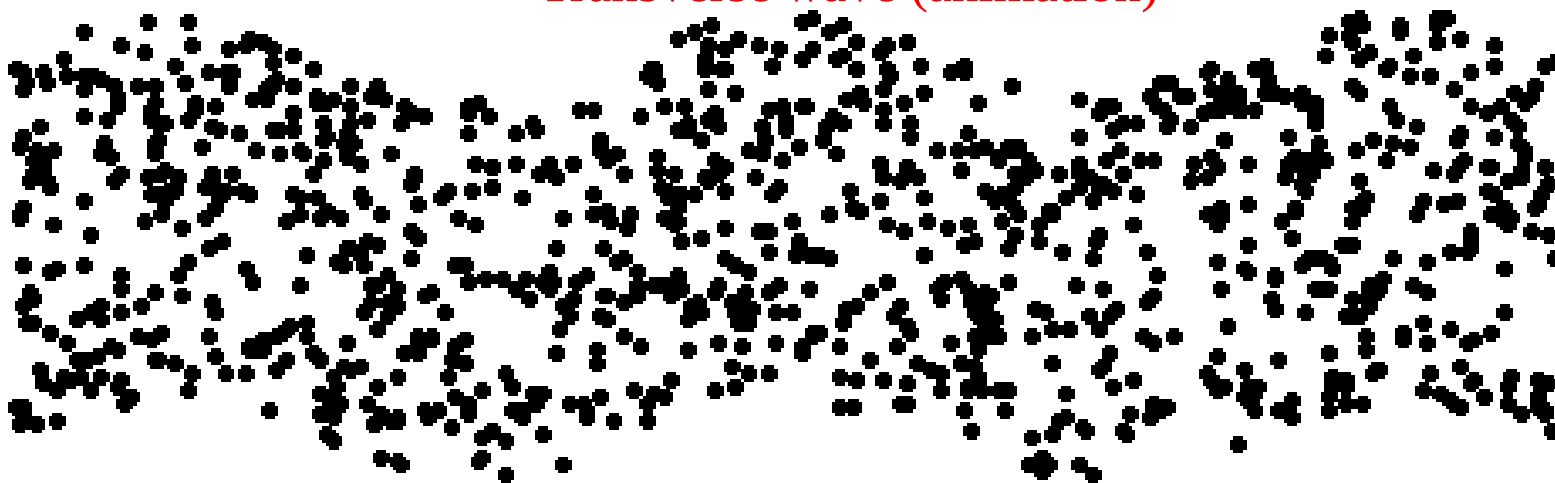
transverse wave in a rope



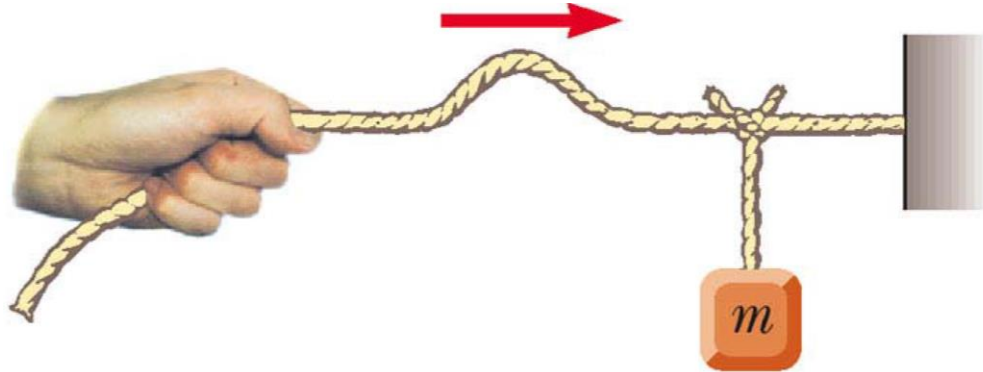
Longitudinal wave (animation)



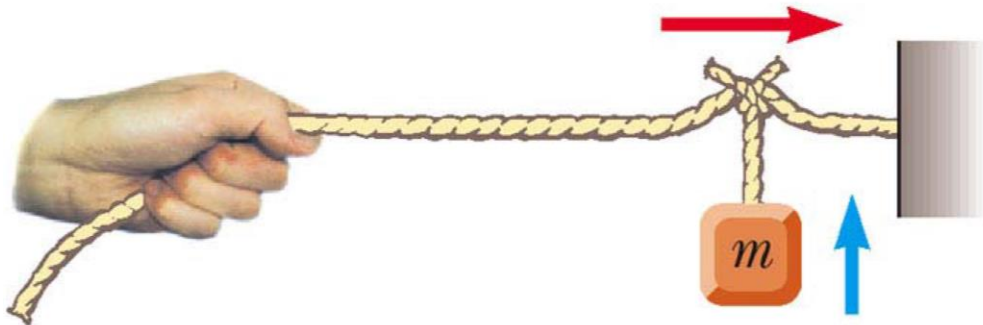
Transverse wave (animation)



Note: In a wave there is a transport of energy
(without “net” mass transport)



(a)



(b)

The fact that the mass m moves upwards (and therefore acquires potential energy) is the “proof” that the wave carries energy.

One-dimensional differential wave equation

Definition: a disturbance $\psi(x, t)$ propagates as a wave without deformation and with a determined v speed if it satisfies:

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

Equation of d'Alembert

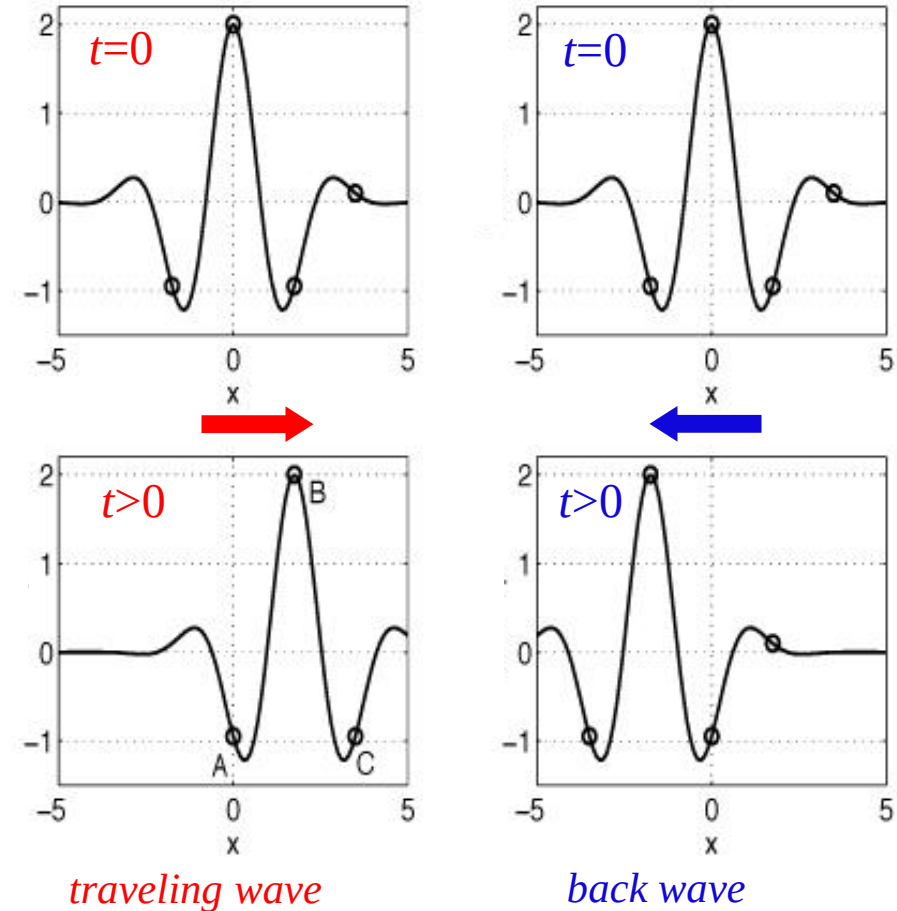
one-dimensional or differential equation of the one-dimensional wave motion

The general solution of the d'Alembert equation is:

$$\psi(x, t) = f(x - vt) + f(x + vt)$$

The d'Alembert wave equation is an example of a linear differential equation, which means that if $\psi_1(x, t)$ and $\psi_2(x, t)$ are solutions to the wave equation, then $\psi_1(x, t) \pm \psi_2(x, t)$ is also a solution.

The implication is that waves solution of the d'Alembert equation (and so also electromagnetic waves, as we will see) obey the superposition principle.



Differential equation of
wave motion
one-dimensional:

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2} \quad \text{or} \quad \frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

General solution:

$$\psi(x, t) = f(x - vt) + f(x + vt)$$

Démonstration:

CURIOSITY

Let's put: $u = x \pm vt$

$\Rightarrow$

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial u} \frac{\partial u}{\partial x} = \frac{\partial \psi}{\partial u}$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial u} \frac{\partial u}{\partial t} = \pm v \frac{\partial \psi}{\partial u}$$

Taking the second derivatives we obtain«

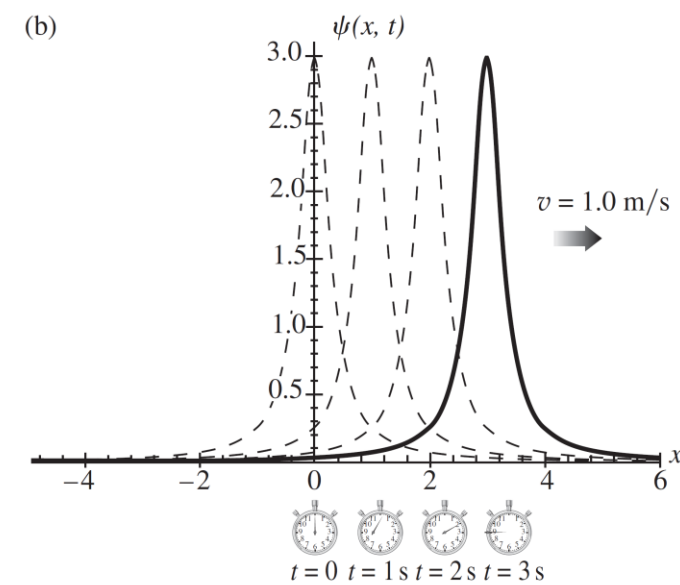
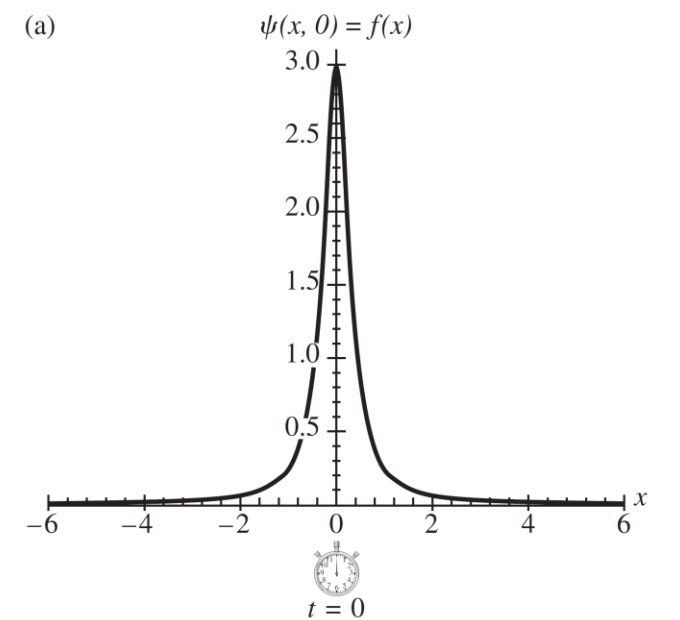
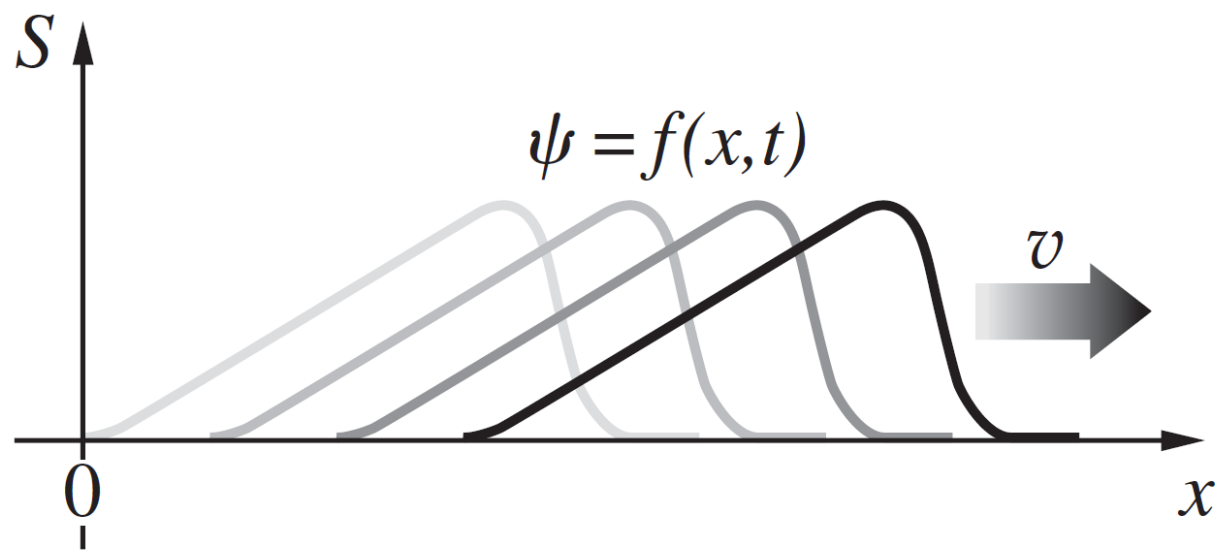
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial u} \frac{\partial u}{\partial x} \right) = \frac{\partial}{\partial u} \left(\frac{\partial \psi}{\partial x} \right) \frac{\partial u}{\partial x} = \frac{\partial^2 \psi}{\partial u^2}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial u} \frac{\partial u}{\partial t} \right) = \frac{\partial}{\partial u} \left(\frac{\partial \psi}{\partial t} \right) \frac{\partial u}{\partial t} = v^2 \frac{\partial^2 \psi}{\partial u^2}$$

$\Rightarrow$

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

Propagation of the disturbance without deformation



The three-dimensional wave differential equation

1D

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

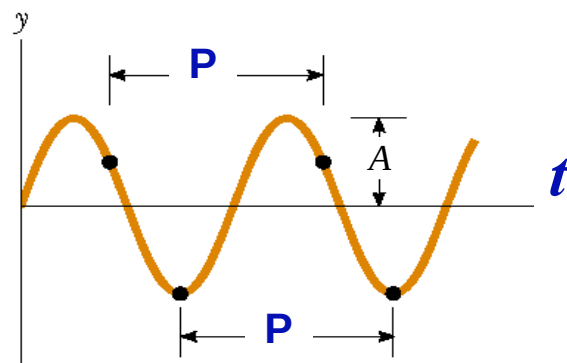
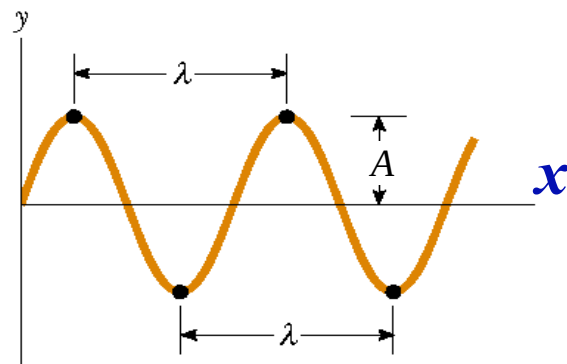
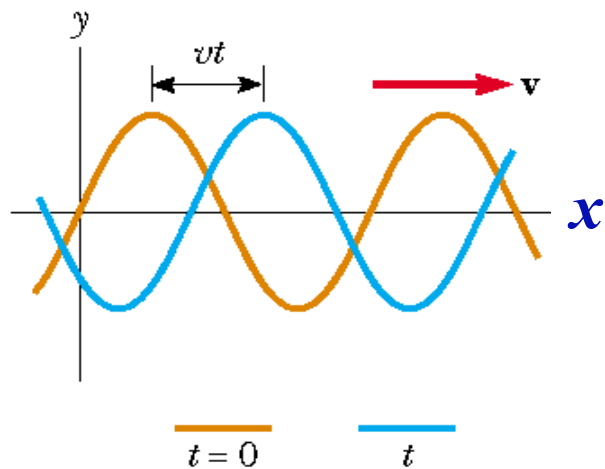
Equation of d'Alembert
one-dimensional or
differential equation of
the wave motion
one-dimensional

3D

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = v^2 \nabla^2 \psi$$

Equation of d'Alembert
three-dimensional or
differential equation of
the wave motion
three-dimensional

Sinusoidal plane wave with propagation along x



Sine wave (equivalent shapes):

$$\psi(x, t) = A \sin[k(x \pm vt)] =$$

$$= A \sin\left(kx \pm \frac{2\pi}{\lambda} vt\right) =$$

$$= A \sin\left(\frac{2\pi}{\lambda} (x \pm vt)\right) =$$

$$= A \sin\left(2\pi\left(\frac{x}{\lambda} \pm \frac{t}{P}\right)\right) =$$

$$= A \sin(kx \pm \omega t) \quad (\text{more common form})$$

$$\lambda \quad \text{Wavelength [m]}$$

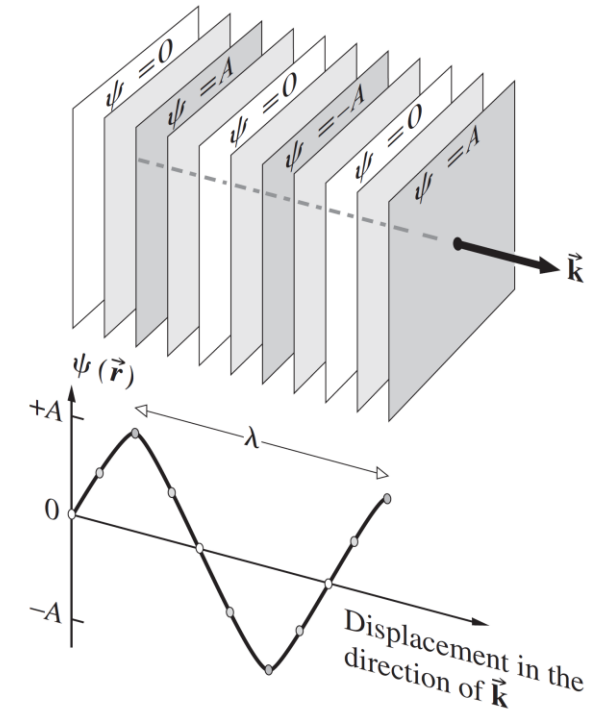
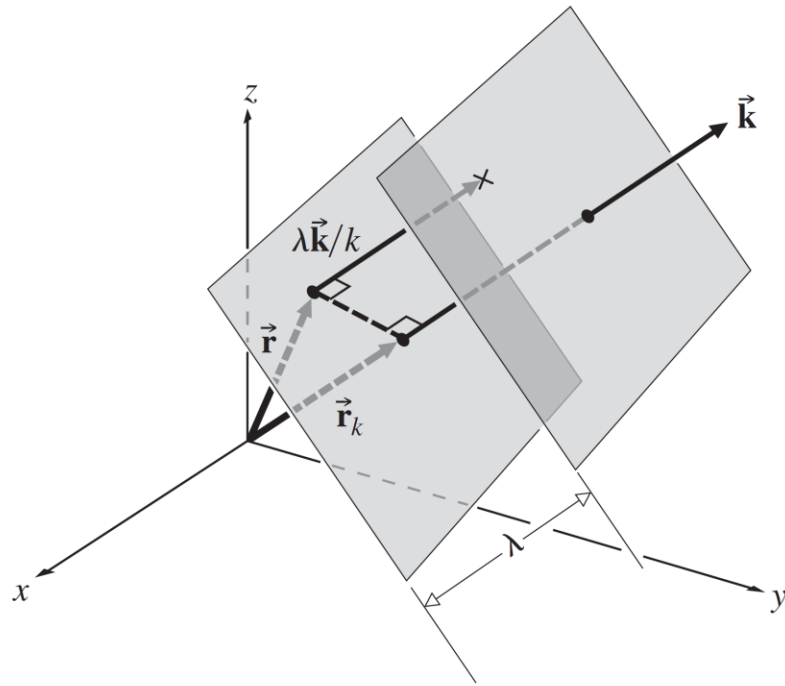
$$k = \frac{2\pi}{\lambda} \quad \text{Wave number [m}^{-1}\text{]}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{P} = \lambda f = \frac{\omega}{k} \quad \text{phase velocity [m/s]}$$

$$f = \frac{v}{\lambda} = \frac{\omega}{2\pi} \quad \text{Frequency [Hz]}$$

$$P = \frac{1}{f} \quad \text{Period [s]}$$

Plane waves with arbitrary propagation direction



Plane sine wave propagating along the axis $\hat{\mathbf{k}}$: $\psi(\mathbf{r}, t) = A \sin(\mathbf{k} \cdot \mathbf{r} \pm \omega t)$

wave-vector $\mathbf{k}$: $\mathbf{k} = k\hat{\mathbf{k}}$ $k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \frac{2\pi}{\lambda} = \frac{\omega}{v}$

$\Rightarrow$

$\psi(\mathbf{r}, t) = A \sin(\mathbf{k} \cdot \mathbf{r} \pm \omega t) = A \sin(k_x x + k_y y + k_z z \pm \omega t)$
 (complex form: $\psi(\mathbf{r}, t) = A e^{i(\mathbf{k} \cdot \mathbf{r} \pm \omega t)}$)

Plane waves are a special case of waves where a physical quantity, such as phase, is constant over a plane that is perpendicular to the direction of wave travel.

Electromagnetic Waves

Relation between Electric and Magnetic fields

- A Change of Magnetic Flux Produces an Electric Field**

The induction of a current in the loop implies the presence of an induced electric field $\vec{E}$, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force.

Kirchhoff's rule:

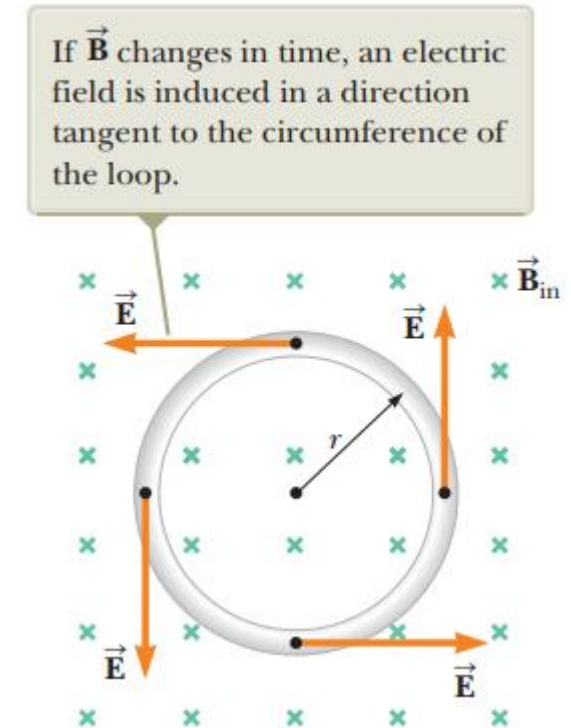
The emf for any closed path can be expressed as the line integral of $\vec{E} \cdot d\vec{\ell}$ over that path

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell}$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



This is a generalization of Faraday's law.

The electric field will exist regardless of whether there are any conductors around.

The induced electric field $\vec{E}$ is a **nonconservative** field that is generated by a changing magnetic field. The field $\vec{E}$ that satisfies the generalization of Faraday's law cannot possibly be an electrostatic field because were the field electrostatic and hence conservative, the line integral over a closed loop would be zero.

Displacement Current and the General Form of Ampère's Law

Can a changing in Electric Fields produce a Magnetic Fields ?

That a magnetic field is produced by an electric current was discovered by Oersted, and the mathematic relation is given by **Ampère's law**

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

In this equation, the line integral is over any closed path through which conduction current passes, where conduction current is defined by the expression $I=dq/dt$.

Is it possible that magnetic fields could be produced in another way as well?

If a changing magnetic field produces an electric field, then perhaps the reverse might be true as well: that a changing electric field will produce a magnetic field.

If this were true, it would signify a beautiful symmetry in nature.

We now show that Ampère's law in this form is valid only if any electric fields present are constant in time.

James Clerk Maxwell recognized this limitation and modified **Ampère's law to include time-varying electric fields**

Displacement Current and the General Form of Ampère's Law

To back up this idea that a changing electric field might produce a magnetic field, we use the following argument:

- According to Ampère's law, we divide any chosen closed path into short segments and take the dot product of each $d\vec{\ell}$ with the magnetic field at that segment $\vec{B}$, and sum (integrate) all these products over the chosen closed path.
- That sum will equal the total current I that passes through a surface bounded by the path of the line integral (I_{encl}).
- When we applied Ampère's law to the field around a straight wire, we imagined the current as passing through the circular area enclosed by our circular loop, and that area is the flat Surface 1 shown in Figure.
- However, we could just as well use the sack-shaped Surface 2 in Figure as the surface for Ampère's law, since the same current I passes through it.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl}$$

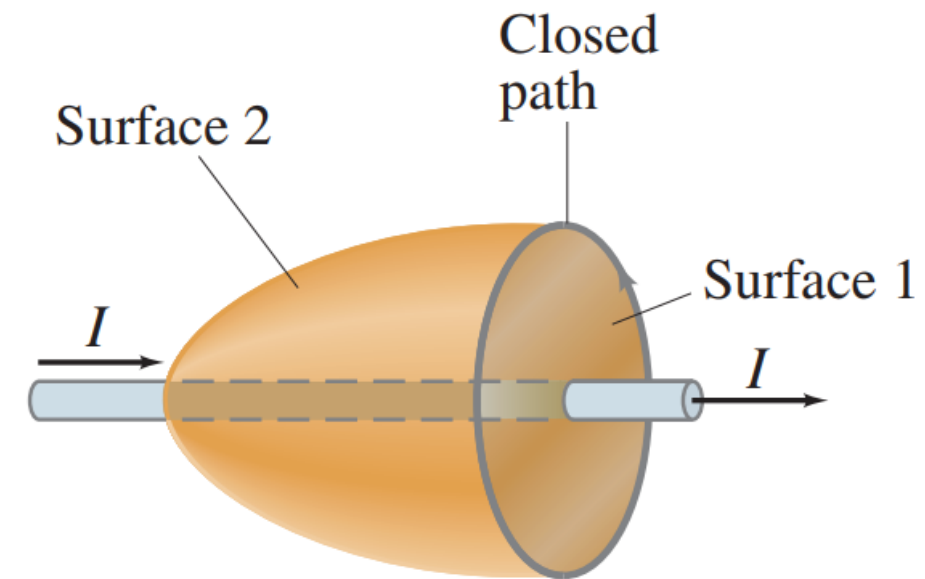


FIGURE 2 Ampère's law applied to two different surfaces bounded by the same closed path.

Displacement Current and the General Form of Ampère's Law

Now consider the closed circular path for the situation of Figure, where a capacitor is being discharged.

Ampère's law works for Surface 1 (current I passes through Surface 1), but it **does not work for Surface 2**, since no current passes through Surface 2.

There is a magnetic field around the wire, so the left side of Ampère's law is not zero; yet no current flows through surface 2, so the right side of Ampère's law is zero.

We seem to have a contradiction of Ampère's law !

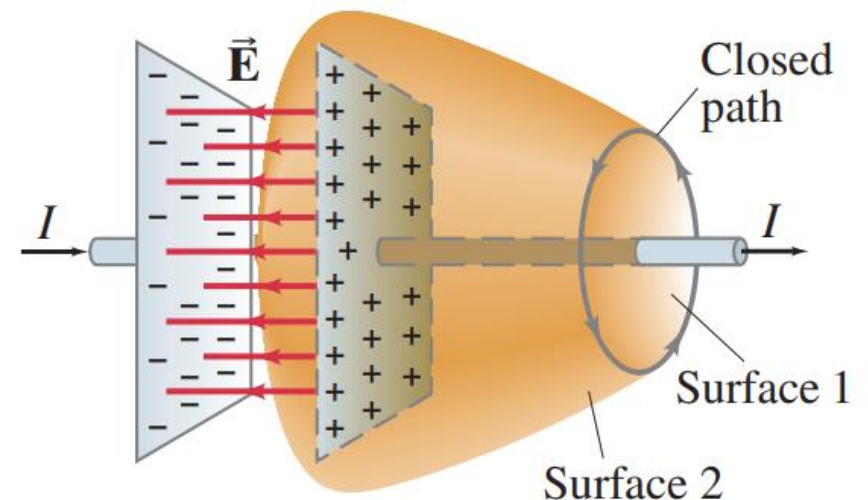
There is a magnetic field present in Figure, however, only if charge is flowing to or away from the capacitor plates.

The changing charge on the plates means that the electric field between the plates is changing in time.

Maxwell resolved the problem of no current through Surface 2 in Figure by proposing that there needs to be **an extra term** on the right in Ampère's law **involving the changing electric field**.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

FIGURE 3 A capacitor discharging. A conduction current passes through surface 1, but no conduction current passes through surface 2. An extra term is needed in Ampère's law.



Displacement Current and the General Form of Ampère's Law

Let us see what this term should be by determining it for the changing electric field between the capacitor plates in Figure.

The charge Q on a capacitor of capacitance C is where V is the potential difference between the plates. Also recall that $V=Ed$, where d is the (small) separation of the plates and E is the (uniform) electric field strength between them, if we ignore any fringing of the field.

Also, for a parallel-plate capacitor, $C = \epsilon_0 A/d$ where A is the area of each plate.

We combine these to obtain

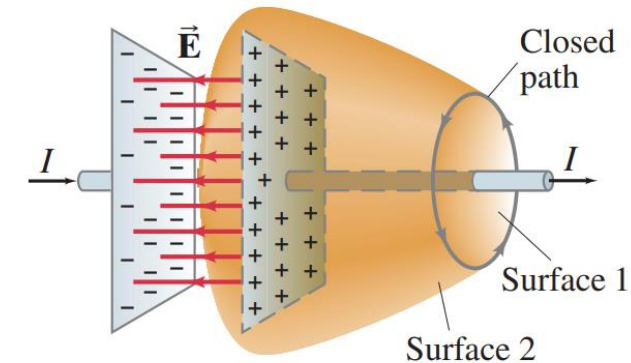
$$Q = CV = \left(\epsilon_0 \frac{A}{d} \right) (Ed) = \epsilon_0 AE$$

If the charge on each plate changes at a rate dQ/dt , the electric field changes at a proportional rate. That is, by differentiating this expression for Q , we have:

Now is also the current I flowing into or out of the capacitor:

$$I = \frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

Where $\Phi_E = EA$ is the electric flux through the closed path (**Surface 2**)

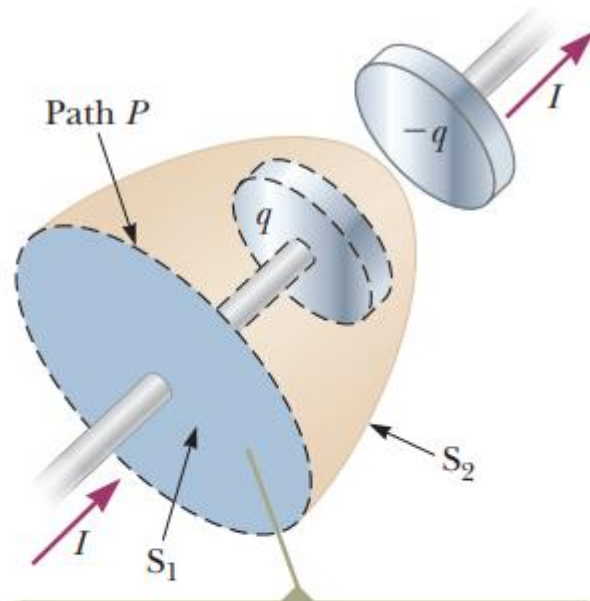


Displacement Current and the General Form of Ampère's Law

In order to make Ampère's law working for surface S2 in Figure, as well as for surface S1 (where current I flows), we therefore write

$I_d =$ displacement current

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad \text{Ampere's law (general form)}$$



The conduction current I in the wire passes only through S₁, which leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S₂.

This equation represents the **general form of Ampère's law**, and embodies Maxwell's idea that **a magnetic field can be caused not only by an ordinary electric current, but also by a changing electric field or changing electric flux.**

Although we arrived at it for a special case, this relation has proved valid in general. The last term on the right of this equation is usually very small, and not easy to measure experimentally

As the capacitor is being charged (or discharged), the changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current in the wire.

Displacement Current and the General Form of Ampère's Law

The electric field lines between the plates create an electric flux through surface S.

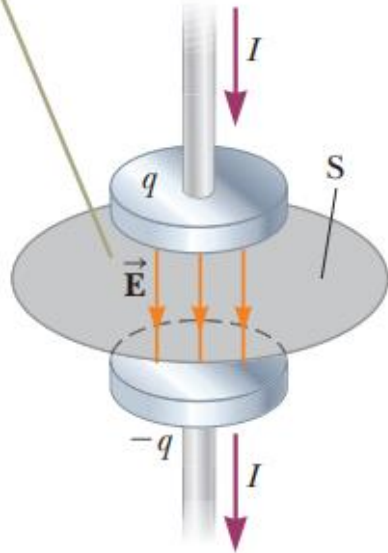


Figure 34.2 When a conduction current exists in the wires, a changing electric field $\vec{E}$ exists between the plates of the capacitor.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

We can understand the meaning of this expression by referring to Figure 34.2. The electric flux through surface S is $\Phi_E = \int \vec{E} \cdot d\vec{A} = EA$, where A is the area of the capacitor plates and E is the magnitude of the uniform electric field between the plates. If q is the charge on the plates at any instant, then $E = q/(\epsilon_0 A)$

Therefore, the electric flux through S is

$$\Phi_E = EA = \frac{q}{\epsilon_0}$$

Hence, the displacement current through S is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt}$$

That is, the displacement current I_d through S is precisely equal to the conduction current I in the wires connected to the capacitor!

EXAMPLE 1 Charging capacitor. A 30-pF air-gap capacitor has circular plates of area $A = 100 \text{ cm}^2$. It is charged by a 70-V battery through a $2.0\text{-}\Omega$ resistor. At the instant the battery is connected, the electric field between the plates is changing most rapidly. At this instant, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume $\vec{E}$ is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

APPROACH In RC circuits, the charge on a capacitor being charged, as a function of time, is

$$Q = CV_0(1 - e^{-t/RC}),$$

where V_0 is the voltage of the battery. To find the current at $t = 0$, we differentiate this and substitute the values $V_0 = 70 \text{ V}$, $C = 30 \text{ pF}$, $R = 2.0 \text{ }\Omega$.

SOLUTION (a) We take the derivative of Q and evaluate it at $t = 0$:

$$\left. \frac{dQ}{dt} \right|_{t=0} = \frac{CV_0}{RC} e^{-t/RC} \Big|_{t=0} = \frac{V_0}{R} = \frac{70 \text{ V}}{2.0 \text{ }\Omega} = 35 \text{ A}.$$

This is the rate at which charge accumulates on the capacitor and equals the current flowing in the circuit at $t = 0$.

(b) The electric field between two closely spaced conductors is given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}.$$

Hence

$$\frac{dE}{dt} = \frac{dQ/dt}{\epsilon_0 A} = \frac{35 \text{ A}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(1.0 \times 10^{-2} \text{ m}^2)} = 4.0 \times 10^{14} \text{ V/m}\cdot\text{s}.$$

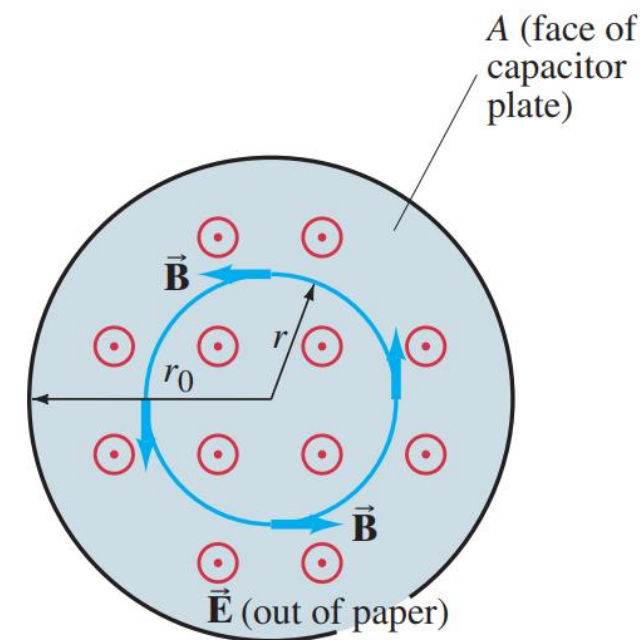


FIGURE 4 Frontal view of a circular plate of a parallel-plate capacitor. $\vec{E}$ between plates points out toward viewer; lines of $\vec{B}$ are circles. (Example 1.)

(c) Although we will not prove it, we might expect the lines of $\vec{B}$, because of *symmetry*, to be circles, and to be perpendicular to $\vec{E}$, as shown in Fig. 4; this is the same symmetry we saw for the inverse situation of a changing magnetic field producing an electric field. To determine the magnitude of B between the plates we apply Ampère's law, Eq. 1, with the current $I_{\text{encl}} = 0$:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

We choose our path to be a circle of radius r , centered at the center of the plate, and thus following a magnetic field line such as the one shown in Fig. 4. For $r \leq r_0$ (the radius of plate) the flux through a circle of radius r is $E(\pi r^2)$ since E is assumed uniform between the plates at any moment. So from Ampère's law we have

$$\begin{aligned} B(2\pi r) &= \mu_0 \epsilon_0 \frac{d}{dt} (\pi r^2 E) \\ &= \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}. \end{aligned}$$

Hence

$$B = \frac{\mu_0 \epsilon_0}{2} r \frac{dE}{dt}. \quad [r \leq r_0]$$

Thus the B field outside the capacitor is the same as that outside the wire. In other words, the magnetic field produced by the changing electric field between the plates is the same as that produced by the current in the wire

$$r > r_0: \quad E = \sigma/\epsilon_0 = Q/(\epsilon_0 A), \quad dE/dt = I/(\epsilon_0 A)$$

$$B = \frac{\mu_0 \epsilon_0 r_0^2}{2r} \frac{dE}{dt} = \frac{\mu_0 \epsilon_0 r_0^2}{2r} \frac{I}{\epsilon_0 \pi r_0^2} = \frac{\mu_0 I}{2\pi r}.$$

We assume $\vec{E} = 0$ for $r > r_0$, so for points beyond the edge of the plates all the flux is contained within the plates (area = πr_0^2) and $\Phi_E = E\pi r_0^2$. Thus Ampère's law gives

$$\begin{aligned} B(2\pi r) &= \mu_0 \epsilon_0 \frac{d}{dt} (\pi r_0^2 E) \\ &= \mu_0 \epsilon_0 \pi r_0^2 \frac{dE}{dt} \end{aligned}$$

or

$$B = \frac{\mu_0 \epsilon_0 r_0^2}{2r} \frac{dE}{dt}. \quad [r \geq r_0]$$

B has its maximum value at $r = r_0$ which, from either relation above (using $r_0 = \sqrt{A/\pi} = 5.6 \text{ cm}$), is

$$\begin{aligned} B_{\text{max}} &= \frac{\mu_0 \epsilon_0 r_0}{2} \frac{dE}{dt} \\ &= \frac{1}{2} (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) (5.6 \times 10^{-2} \text{ m}) (4.0 \times 10^{14} \text{ V/m} \cdot \text{s}) \\ &= 1.2 \times 10^{-4} \text{ T}. \end{aligned}$$

This is a very small field and lasts only briefly (the time constant $RC = 6.0 \times 10^{-11} \text{ s}$) and so would be very difficult to measure.

Gauss's Law in Magnetism (closed surface)

The magnetic field lines generated by a current and of a bar magnet do not begin or end at any point. **For any closed surface, the number of lines entering the surface equals the number leaving the surface; therefore, the net magnetic flux is zero.** In contrast, for a closed surface surrounding one charge of an electric, the net electric flux is not zero.

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

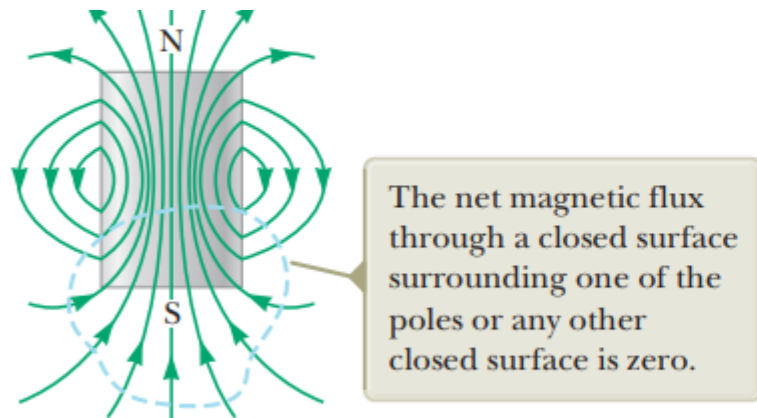


Figure 30.22 The magnetic field lines of a bar magnet form closed loops. (The dashed line represents the intersection of a closed surface with the page.)

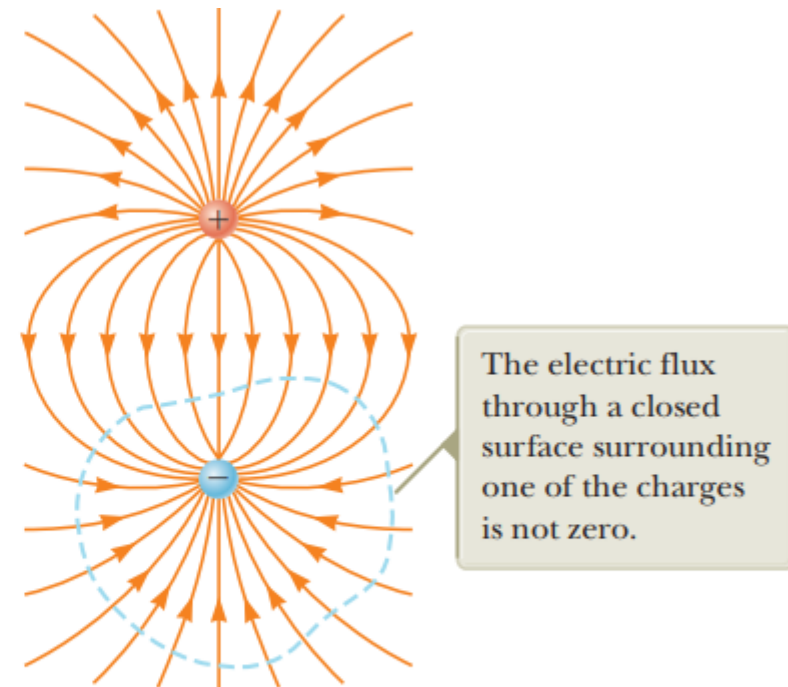


Figure 30.23 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge.

Maxwell's Equations (integral form)

We now present **four equations** that are regarded as the basis of all electrical and magnetic phenomena. These equations, developed by Maxwell, are as fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena.

The theory that Maxwell developed turned out to also be in agreement with the special theory of relativity, as Einstein showed in 1905.

Maxwell's equations represent the laws of electricity and magnetism that we have already discussed, but they have additional important consequences. For simplicity, we present Maxwell's equations as applied to free space, that is, in the absence of any dielectric or magnetic material. The four equations are:

$$(1) \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

◀ Gauss's law

$$(2) \quad \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

◀ Gauss's law in magnetism

$$(3) \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

◀ Faraday's law

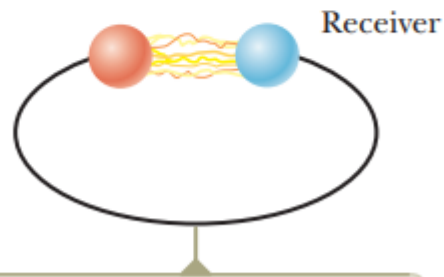
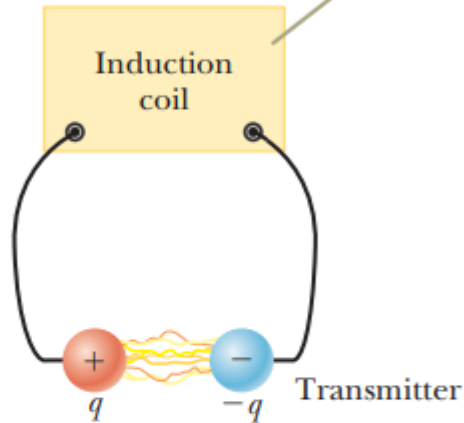
$$(4) \quad \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

◀ Ampère–Maxwell law

Notice the symmetry of Maxwell's equations. Equations (1) and (2) are symmetric, apart from the absence of the term for magnetic monopoles in Equation (2). Furthermore, Equations (3) and (4) are symmetric in that the line integrals of $\mathbf{E}$ and $\mathbf{B}$ around a closed path are related to the rate of change of magnetic flux and electric flux, respectively.

Hertz's Discoveries

The transmitter consists of two spherical electrodes connected to an induction coil, which provides short voltage surges to the spheres, setting up oscillations in the discharge between the electrodes.

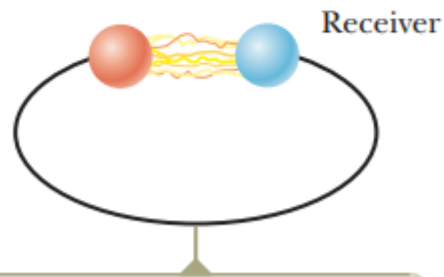
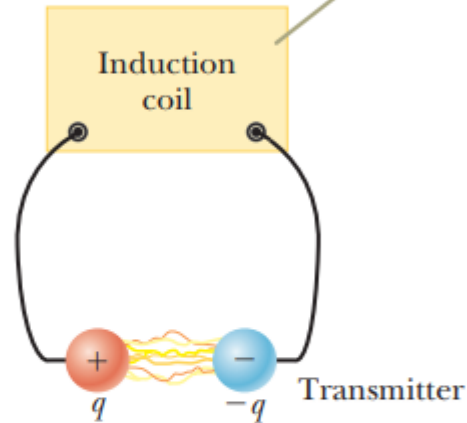


The receiver is a nearby loop of wire containing a second spark gap.

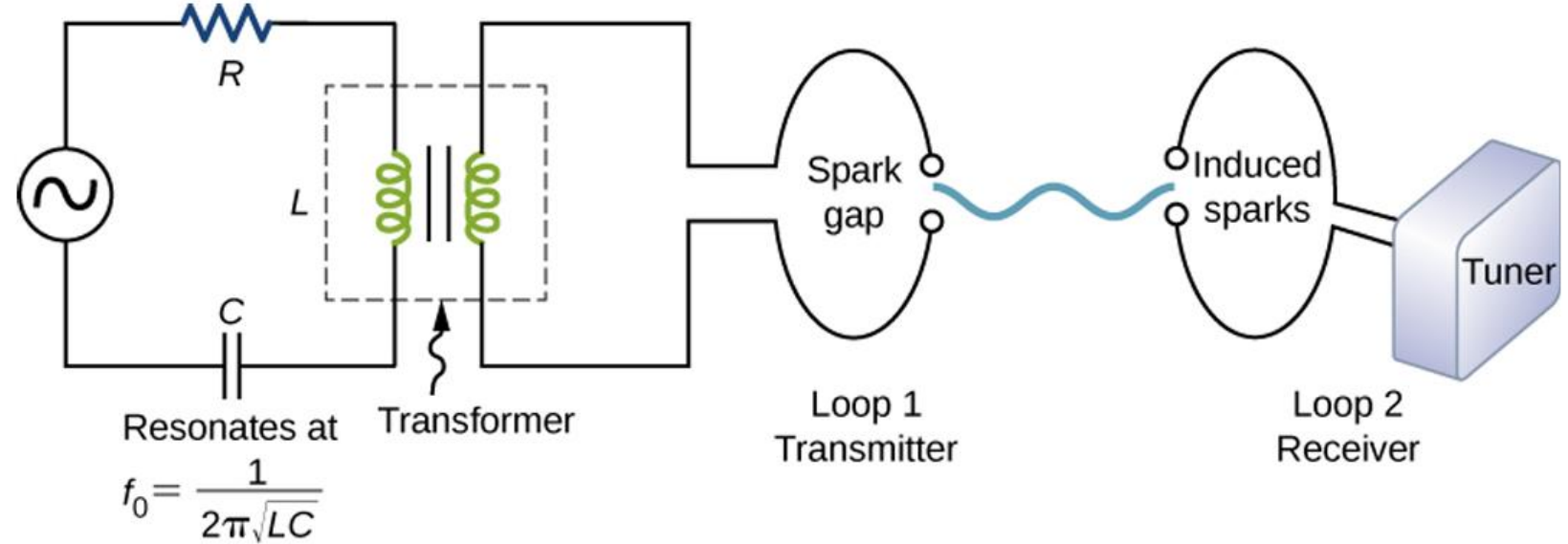
- Hertz performed experiments that verified Maxwell's prediction. The experimental apparatus Hertz used to generate and detect electromagnetic waves is shown schematically in Figure.
- An induction coil is connected to a **transmitter** made up of two spherical electrodes separated by a narrow gap. The coil provides short voltage surges to the electrodes, making one positive and the other negative. A spark is generated between the spheres when the electric field near either electrode surpasses the dielectric strength for air (3×10^6 V/m).
- From an electric-circuit viewpoint, this experimental apparatus is equivalent to an LC circuit in which the inductance is that of the coil and the capacitance is due to the spherical electrodes.

Hertz's Discoveries

The transmitter consists of two spherical electrodes connected to an induction coil, which provides short voltage surges to the spheres, setting up oscillations in the discharge between the electrodes.



The receiver is a nearby loop of wire containing a second spark gap.



The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves.

Because L and C are small in Hertz's apparatus, the frequency of oscillation is high, on the order of 100 MHz. **Electromagnetic waves are radiated at this frequency as a result of the oscillation of free charges in the transmitter circuit.**

Hertz was able to detect these waves using a single loop of wire with its own spark gap (the receiver). Such a **receiver** loop, placed several meters from the transmitter, has its own effective inductance, capacitance, and natural frequency of oscillation.

In Hertz's experiment, sparks were induced across the gap of the receiving electrodes when the receiver's frequency was adjusted to match that of the transmitter.

Production of Electromagnetic Waves

A magnetic field will be produced in empty space if there is a changing electric field.
A changing magnetic field produces an electric field that is itself changing.
This changing electric field will, in turn, produce a magnetic field, which will be changing, and so it too will produce a changing electric field; and so on.

Maxwell found that the net result of these interacting changing fields was a wave of electric and magnetic fields that can propagate (travel) through space!

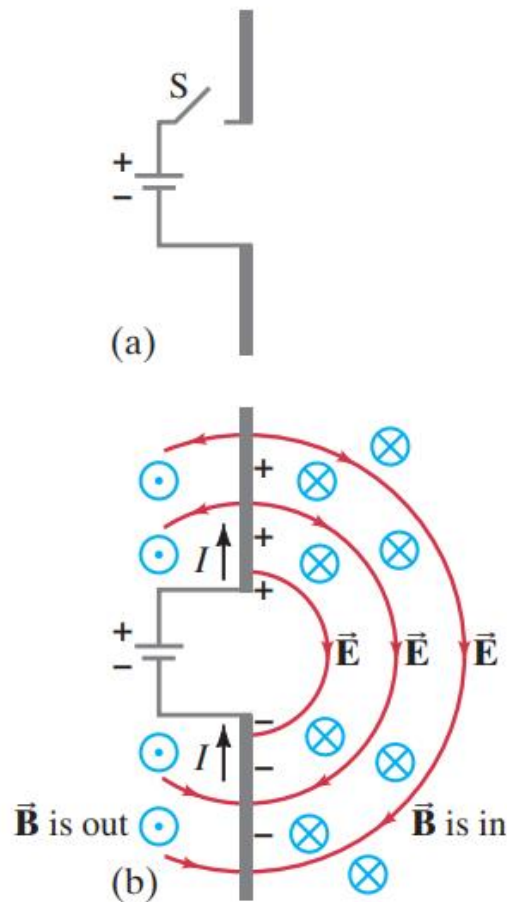


FIGURE 6 Fields produced by charge flowing into conductors. It takes time for the $\vec{E}$ and $\vec{B}$ fields to travel outward to distant points. The fields are shown to the right of the antenna, but they move out in all directions, symmetrically about the (vertical) antenna.

- Consider two conducting rods that will serve as an “antenna”. Suppose these two rods are connected by a switch to the opposite terminals of a battery. When the switch is closed, the upper rod quickly becomes positively charged and the lower one negatively charged.
 - Electric field lines are formed as indicated in Figure. While the charges are flowing, a current exists whose direction is indicated by the black arrows. A magnetic field is therefore produced near the antenna. The magnetic field lines encircle the rod-like antenna and therefore points into the page on the right and out of the page on the left.
 - In the static case, the fields extend outward indefinitely far.
- However, when the switch is closed, the fields quickly appear nearby, but it takes time for them to reach distant points. Both electric and magnetic fields store energy, and this energy cannot be transferred to distant points at infinite speed.

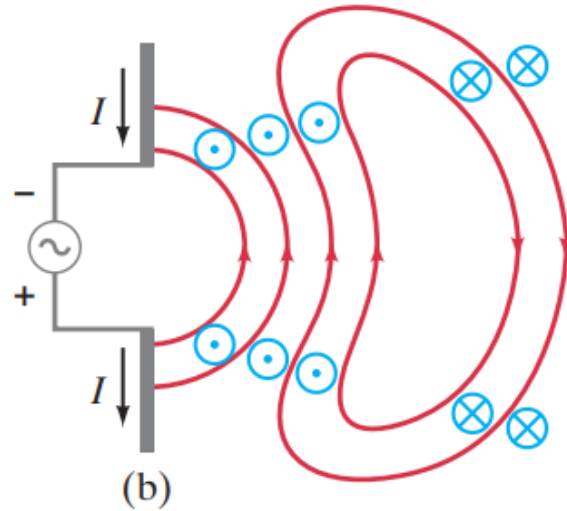
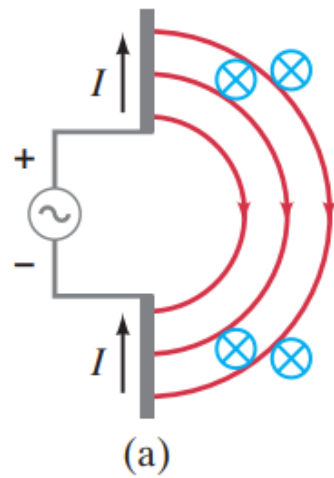


FIGURE 7 Sequence showing electric and magnetic fields that spread outward from oscillating charges on two conductors (the antenna) connected to an ac source (see the text).

Now we look at a different situation, where our **antenna is connected to an ac generator.**

In Fig. a, the connection has just been realized. Charge starts building up and fields form.

The + and - signs in Fig. a indicate the net charge on each rod at a given instant.

The black arrows indicate the direction of the current I . The electric field is represented by the red lines in the plane of the page; and the magnetic field, according to the right-hand rule, is into or out of the page, in blue

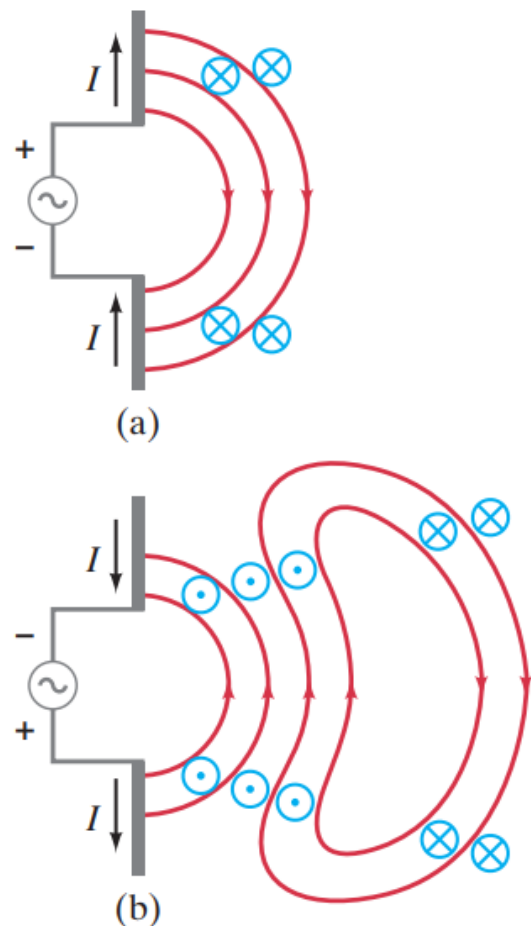
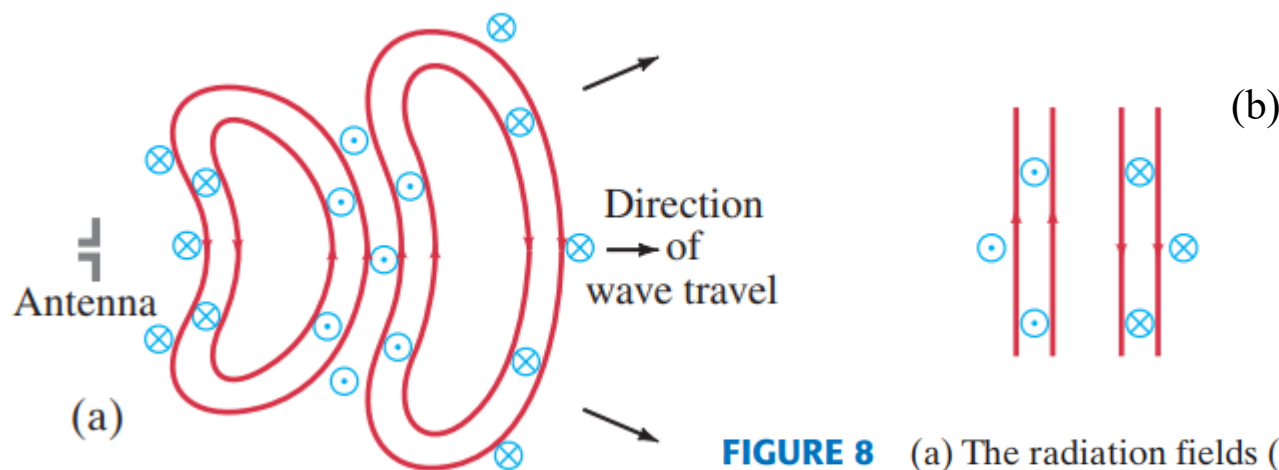


FIGURE 7 Sequence showing electric and magnetic fields that spread outward from oscillating charges on two conductors (the antenna) connected to an ac source (see the text).

In Fig. b, the voltage of the ac generator has reversed in direction; the current is reversed, and the new magnetic field is in the opposite direction. Because the new fields have changed direction, the **old lines fold back** to connect up to some of the new lines **and form closed loops**, as shown.

The old fields, however, don't suddenly disappear; they are on their way to distant points. Indeed, because a changing magnetic field produces an electric field, and a changing electric field produces a magnetic field, **this combination of changing electric and magnetic fields moving outward is self-supporting**, no longer depending on the antenna charges.



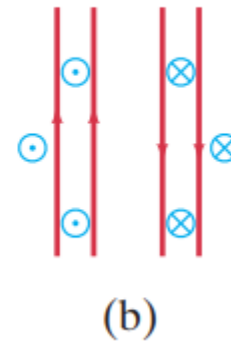
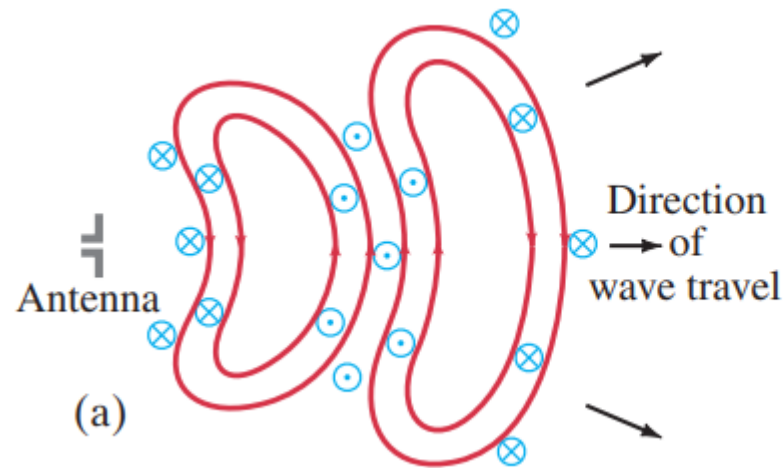
SELF SUPPORTING WAVES

FIGURE 8 (a) The radiation fields (far from the antenna) produced by a sinusoidal signal on the antenna. The red closed loops represent electric field lines. The magnetic field lines, perpendicular to the page and represented by blue $\otimes$ and $\odot$, also form closed loops. (b) Very far from the antenna the wave fronts (field lines) are essentially flat over a fairly large area, and are referred to as *plane waves*.

The fields *not far* from the antenna, referred to as the *near field*, become quite complicated.

We are instead mainly interested in the fields *far from the antenna*, which we refer to as the *radiation field, or far field*. The electric field lines form loops, as shown in Fig., and continue moving outward.

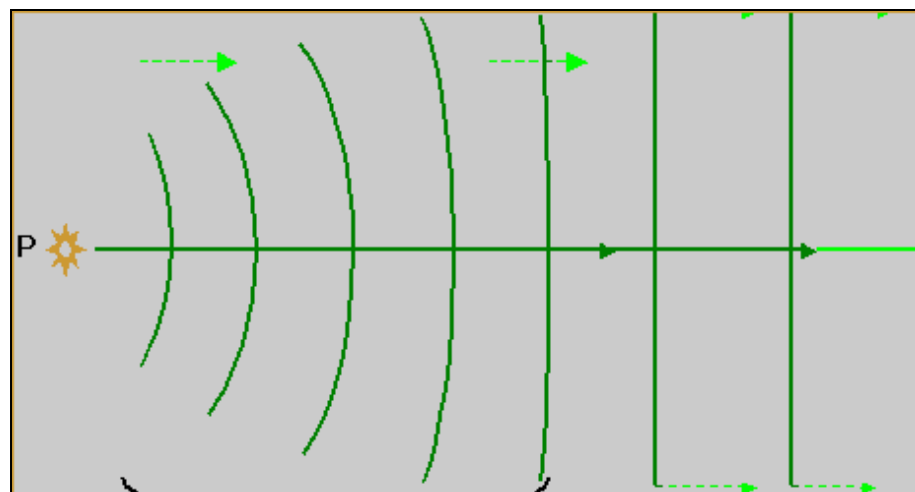
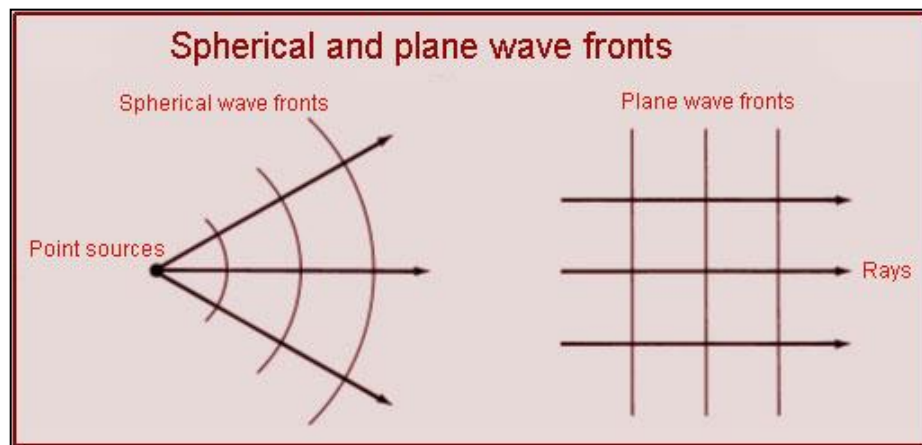
The magnetic field lines also form closed loops but are not shown since they are perpendicular to the page.



Several things about the radiation field can be noted from this Fig..

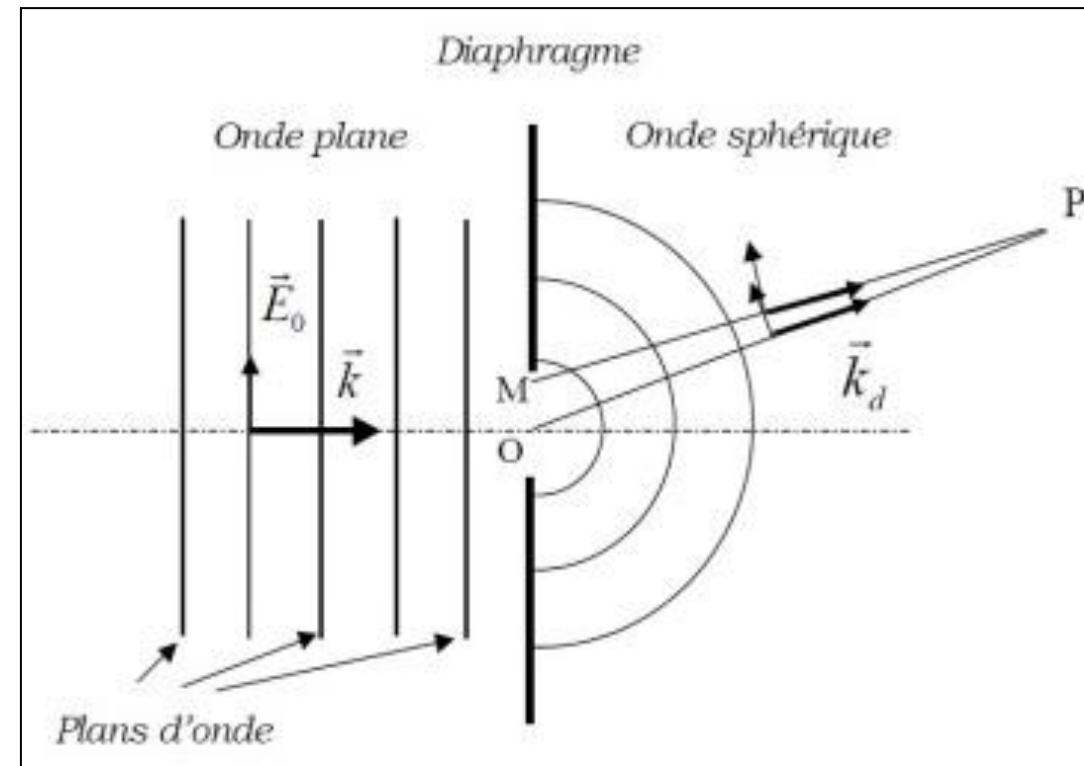
- (1) the electric and magnetic fields at any point are perpendicular to each other, and to the direction of wave travel.
- (2) we can see that the fields alternate in direction ($\mathbf{B}$ is into the page at some points and out of the page at others; $\mathbf{E}$ points up at some points and down at others). Thus, the field strengths vary from a maximum in one direction, to zero, to a maximum in the other direction.
- (3) the electric and magnetic fields are “in phase”: that is, they each are zero at the same points and reach their maxima at the same points in space.
- (4) very far from the antenna the field lines are quite flat over a reasonably large area, and the waves are referred to as **plane waves**.

Plane and spherical waves



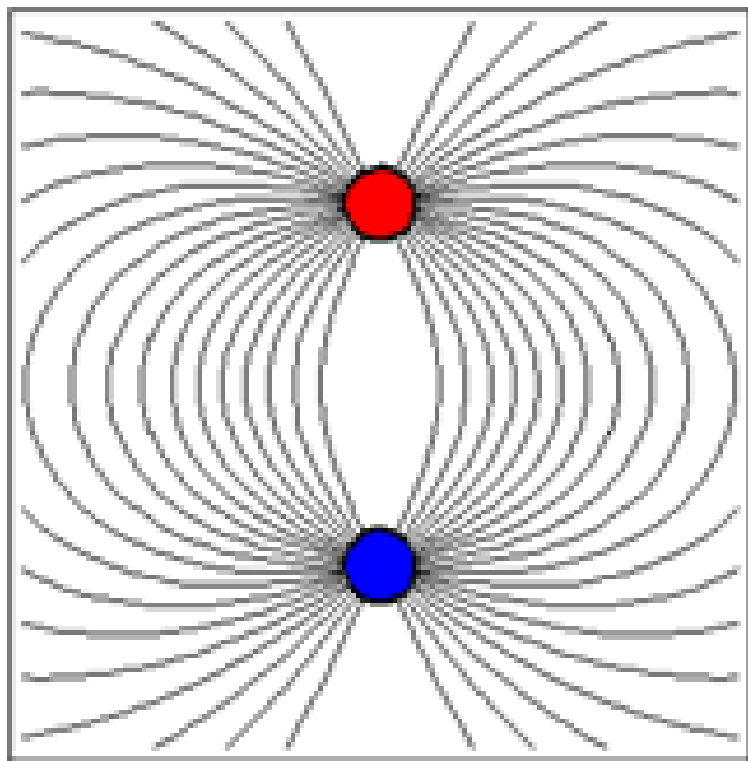
Spherical
(near source)

Plane
(far from source)



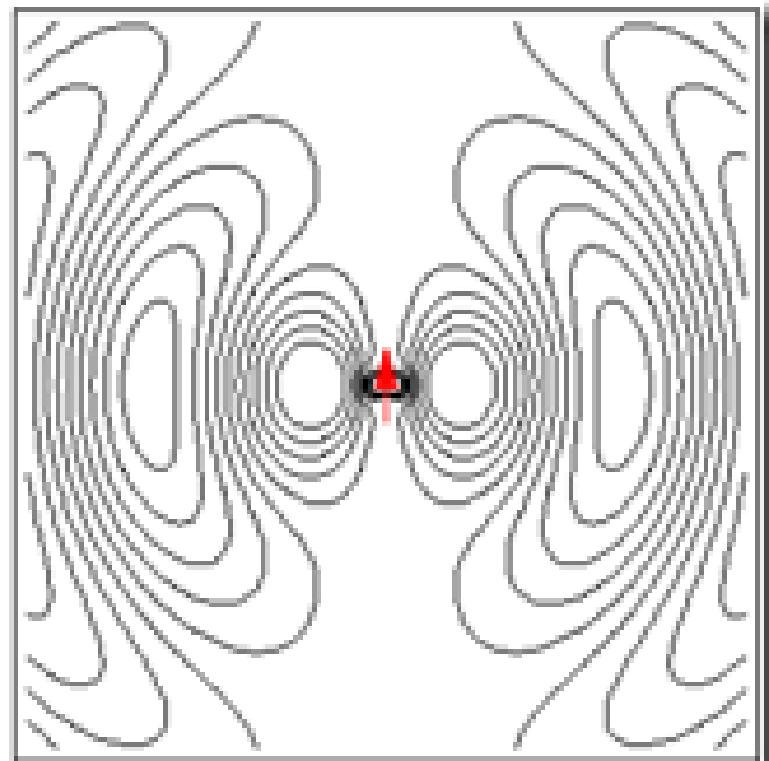
Static Electric Dipole

(generates a static electric field)

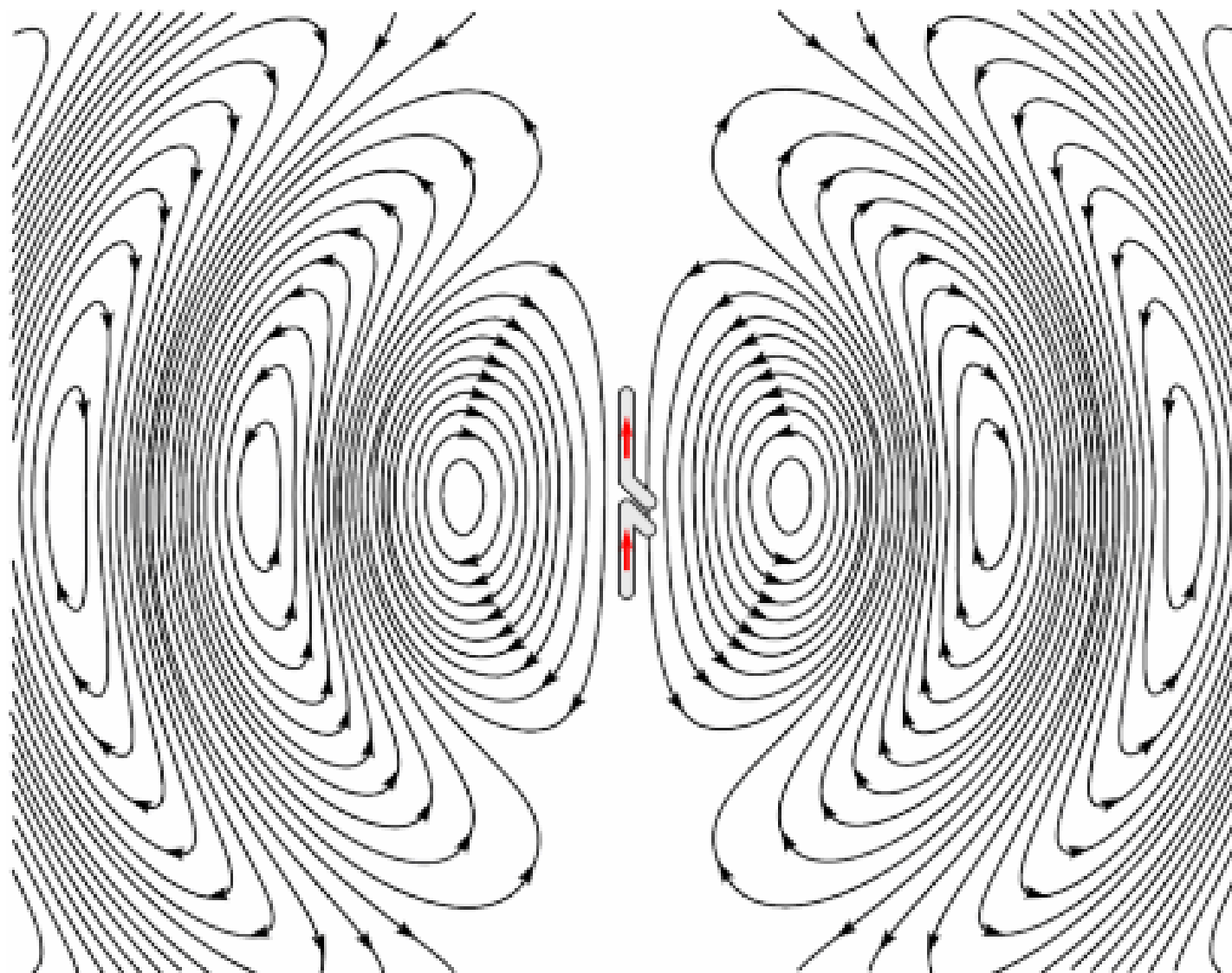


Oscillating Electric Dipole

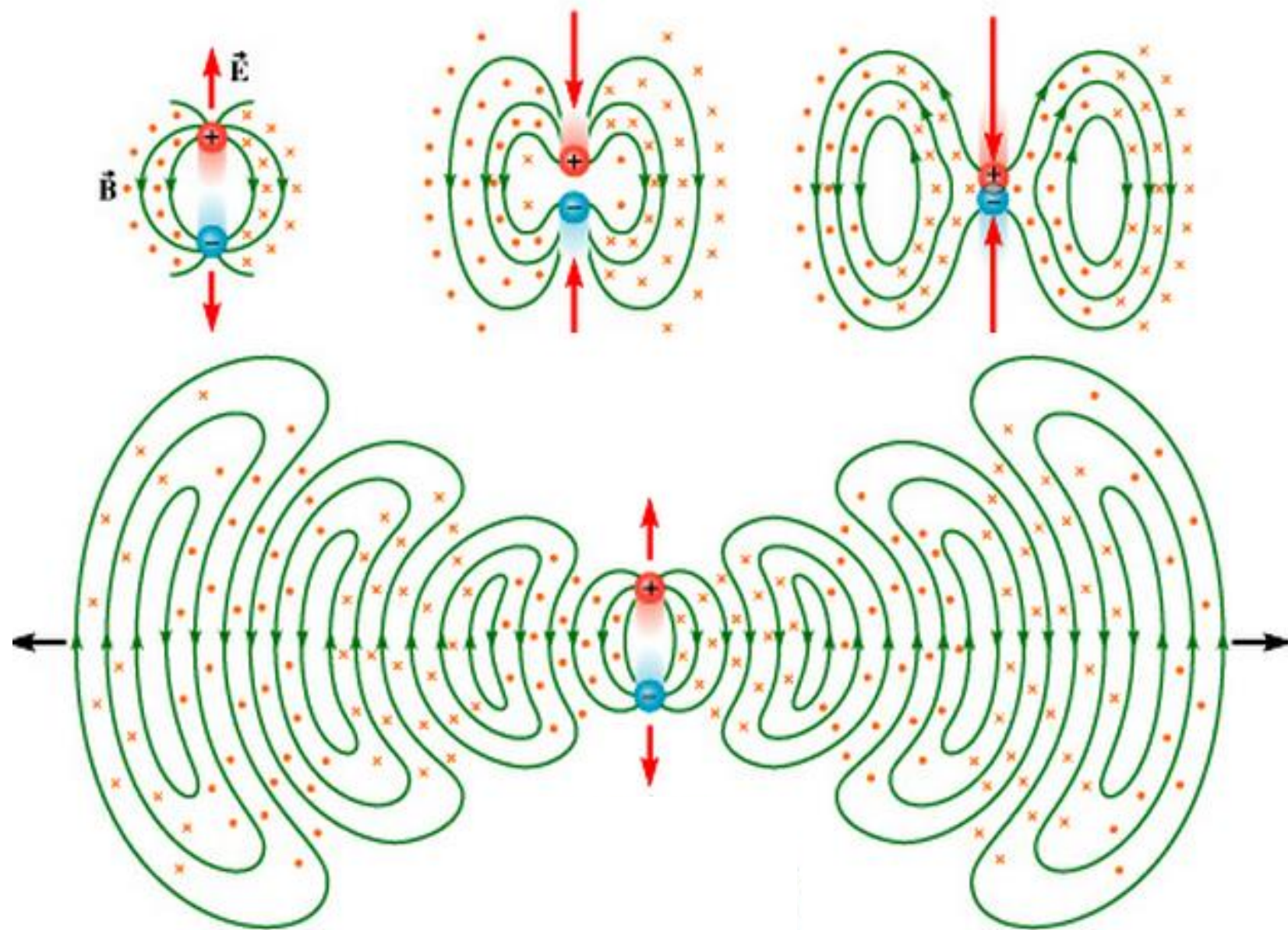
(generates an oscillating electric field
and
an oscillating magnetic field)



Field created by an oscillating electric dipole



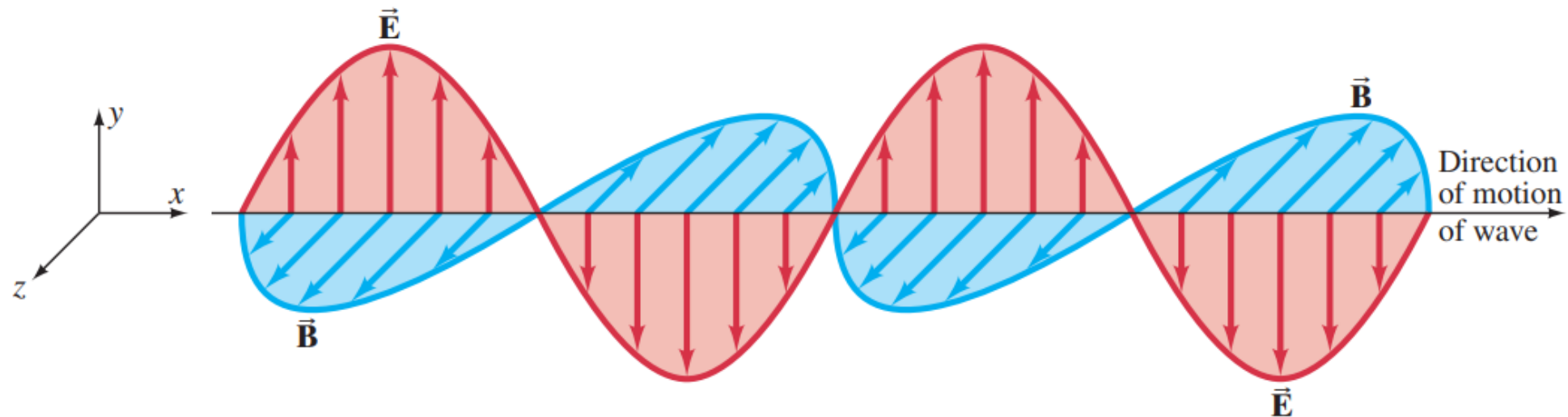
(animation)



Production of Electromagnetic Waves

If the source voltage varies sinusoidally, then the electric and magnetic field strengths in the radiation field will also vary sinusoidally. The sinusoidal character of the waves is diagrammed in Fig., which shows the field directions and magnitudes plotted as a function of position.

Notice that **B** and **E** are perpendicular to each other and to the direction of travel (=the direction of the wave velocity $\mathbf{v}$). The direction of $\mathbf{v}$ can be found from a right-hand rule using $\mathbf{E} \times \mathbf{B}$.



- We call these waves **electromagnetic (EM) waves**. They are **transverse waves** because the amplitude is perpendicular to the direction of wave travel.
- **EM waves are always waves of fields, not of matter** (like waves on water or a rope).
- Because they are fields, EM waves can propagate **in empty space**.
- **EM waves are produced by electric charges that are oscillating and hence are undergoing acceleration.**
Accelerating electric charges give rise to electromagnetic waves.

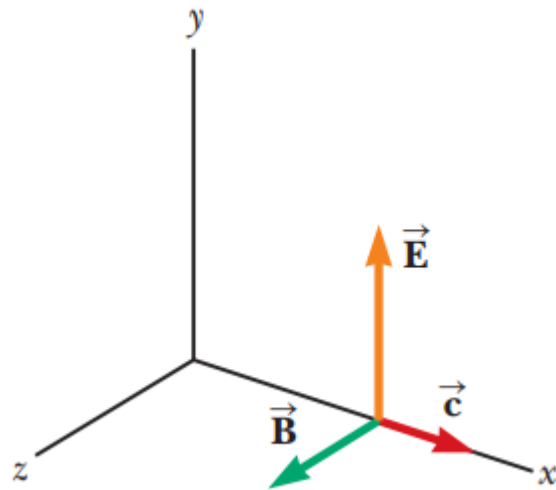
Electromagnetic Waves in vacuum, and Their Speed, Derived from Maxwell's Equations

Let us now examine how the existence of EM waves follows from Maxwell's equations.

We begin by considering a **region of free space**, where there are **no charges or conduction currents**—that is, far from the source so that the wave fronts are essentially flat over a reasonable area.

We call them plane waves, as we saw, because at any instant **B** and **E** are uniform over a reasonably large plane perpendicular to the direction of propagation.

We choose a coordinate system, so that the wave is traveling in the x direction with velocity **v** with **E** parallel to the y axis and **B** parallel to the z axis.



Maxwell's equations in vacuum, with $Q=I=0$ (no sources), become:

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

Maxwell's equations, with $Q=I=0$:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = - \frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

Notice the **beautiful symmetry** of these equations. The term on the right in the last equation, conceived by Maxwell, is essential for this symmetry. It is also essential if electromagnetic waves are to be produced, as we will now see.

If the wave is sinusoidal with wavelength λ and frequency f , then such a traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad \text{and} \quad f\lambda = \frac{\omega}{k} = v, \quad \text{speed of the wave}$$

Such waves, in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes, are said to be **linearly polarized waves**.

Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

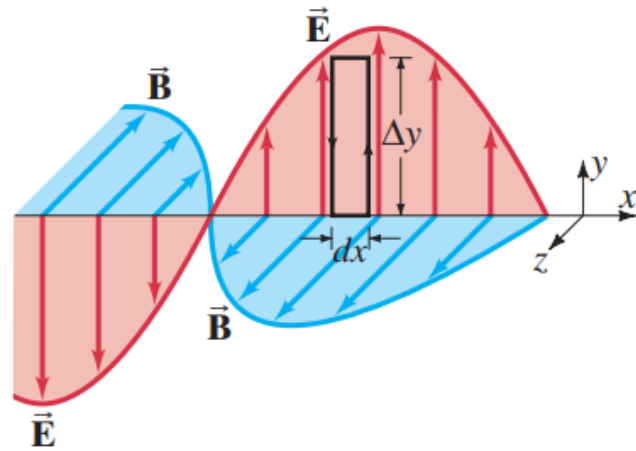


FIGURE 10 Applying Faraday's law to the rectangle $(\Delta y)(dx)$.

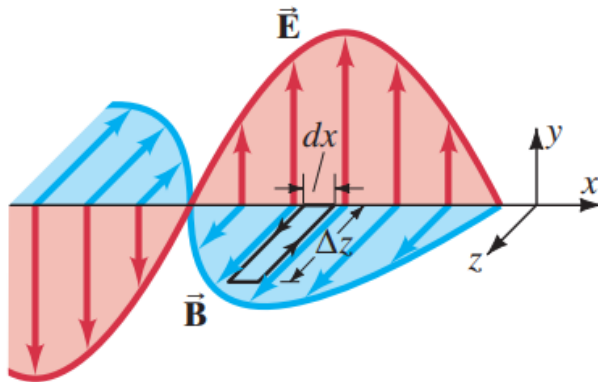


FIGURE 11 Applying Maxwell's fourth equation to the rectangle $(\Delta z)(dx)$.

According to Equation 34.11, this spatial variation in $\vec{E}$ gives rise to a time-varying magnetic field along the z direction.

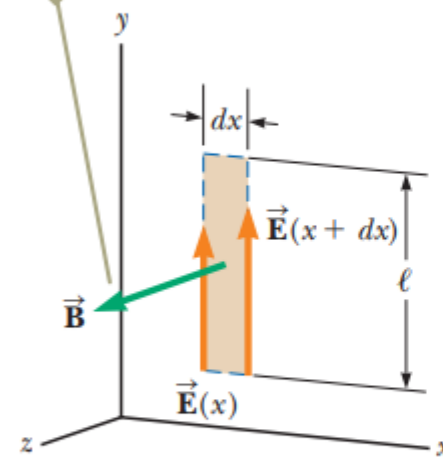


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x+dx)$.

According to Equation 34.14, this spatial variation in $\vec{B}$ gives rise to a time-varying electric field along the y direction.

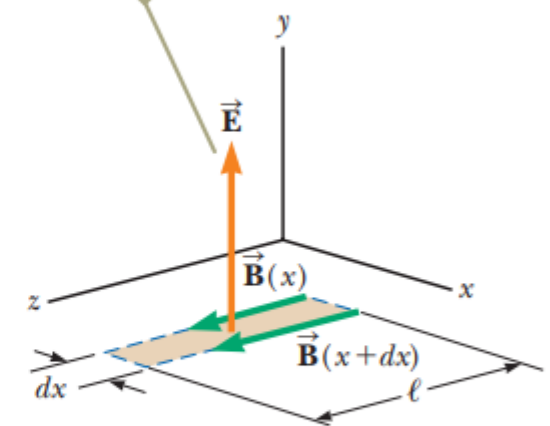


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{B}(x)$ to $\vec{B}(x+dx)$.

Let's consider those circuits and let's apply Faraday's law and Ampere/Maxwell's law.

Electromagnetic Waves, and Their Speed, Derived from Maxwell's Equations

$\Phi_B = B\ell dx$ assuming dx is very small compared with the wavelength of the wave

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law applied to the rectangle in Figure

Consider a rectangle of width dx and height ℓ , lying in the xy plane as shown in Figure. Let's first evaluate the line integral of $\vec{E} \cdot d\vec{s}$ around this rectangle in the counterclockwise direction at an instant of time when the wave is passing through the rectangle.

The contributions from the top and bottom of the rectangle are zero because $\vec{E}$ is perpendicular to $d\vec{s}$ for these paths.

We can express the electric field on the right side of the rectangle as

$$E(x + dx) \approx E(x) + \left. \frac{dE}{dx} \right|_{t \text{ constant}} dx = E(x) + \frac{\partial E}{\partial x} dx \quad \Rightarrow$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{s} = [E(x + dx)]\ell - [E(x)]\ell \approx \ell \left(\frac{\partial E}{\partial x} \right) dx \quad \left. \begin{aligned} &\ell \left(\frac{\partial E}{\partial x} \right) dx = -\ell dx \frac{\partial B}{\partial t} \\ &\frac{d\Phi_B}{dt} = \ell dx \frac{dB}{dt} \Big|_{x \text{ constant}} = \ell dx \frac{\partial B}{\partial t} \end{aligned} \right\} \quad \boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

According to Equation 34.11, this spatial variation in $\vec{E}$ gives rise to a time-varying magnetic field along the z direction.

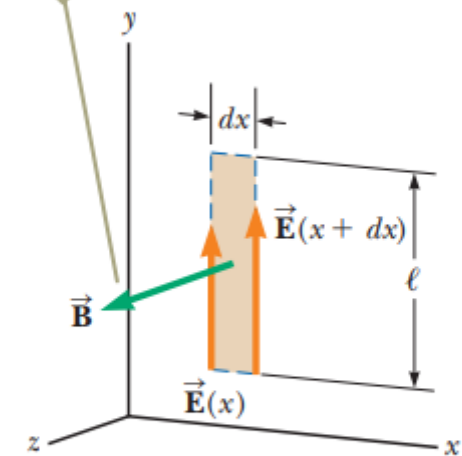


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x + dx)$.

Noting that the magnitude of the magnetic field changes from $B(x)$ to $B(x+dx)$ over the width dx and that the direction for taking the line integral is counterclockwise when viewed from above in Figure

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = [B(x)]\ell - [B(x+dx)]\ell \approx -\ell \left(\frac{\partial B}{\partial x} \right) dx$$

$$\Phi_E = E\ell dx, \quad \frac{\partial \Phi_E}{\partial t} = \ell dx \frac{\partial E}{\partial t}$$

$$-\ell \left(\frac{\partial B}{\partial x} \right) dx = \mu_0 \epsilon_0 \ell dx \left(\frac{\partial E}{\partial t} \right)$$

$$\boxed{\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$$

Ampère's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \cancel{\mu_0 I} + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

According to Equation 34.14, this spatial variation in $\vec{\mathbf{B}}$ gives rise to a time-varying electric field along the y direction.

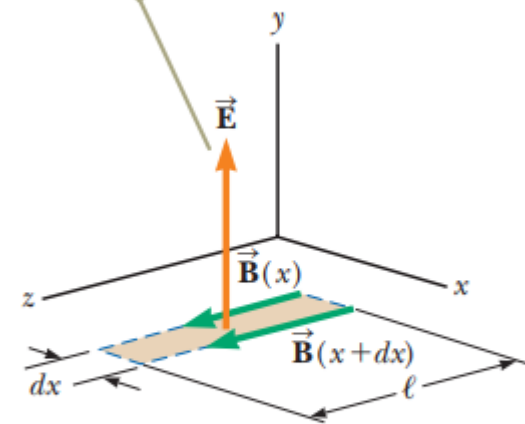


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{\mathbf{B}}(x)$ to $\vec{\mathbf{B}}(x+dx)$.

Slide 13.38

Slide 13.39

Consider

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

and

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{\partial}{\partial x} \Rightarrow \frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x} \left(\frac{\partial B}{\partial t} \right) = -\frac{\partial}{\partial t} \left(\frac{\partial B}{\partial x} \right) = -\frac{\partial}{\partial t} \left(-\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right) \Rightarrow \frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Remember: Equation of
(slide 13.6) **d'Alembert**

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

In a similar way, one
can show that it is
possible to obtain:

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

These equations both have the form of the linear wave
equation with the wave speed v replaced by c , where:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$c = \frac{1}{\sqrt{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(8.854\,19 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}} \\ = 2.997\,92 \times 10^8 \text{ m/s}$$

Because this speed is precisely the same as the
speed of light in empty space, we are led to
believe (correctly) that
light is an electromagnetic wave !!!

The simplest solution of those wave equations is a sinusoidal wave for which the field magnitudes E and B vary with x and t according to the expressions:

$$(1) \quad E = E_{\max} \cos(kx - \omega t)$$

$$(2) \quad B = B_{\max} \cos(kx - \omega t)$$

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

where $E_{\max}$ and $B_{\max}$ are the maximum values of the fields.

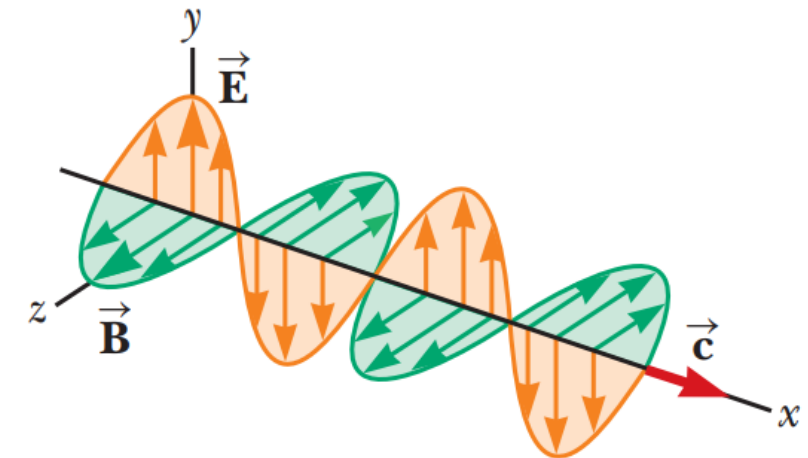
The angular wave number is $k = \frac{2\pi}{\lambda}$, where λ is the wavelength.

The angular frequency is $\omega = 2\pi f$, where f is the wave frequency. According to the traveling wave model, the ratio ω/k equals the speed of an electromagnetic wave, c

where we have used $v = c = \lambda f$, which relates the speed, frequency, and wavelength of a sinusoidal wave.

Therefore, for electromagnetic waves, the wavelength and frequency of these waves are related by

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f}$$



Pictorial representation, at one instant, of a sinusoidal, linearly polarized electromagnetic wave moving in the positive x direction.

Taking partial derivatives of Equation (1) (with respect to x) and (2) (with respect to t) gives

$$\begin{aligned}
 (1) \quad E &= E_{\max} \cos(kx - \omega t) & \frac{\partial E}{\partial x} &= -kE_{\max} \sin(kx - \omega t) \\
 (2) \quad B &= B_{\max} \cos(kx - \omega t) & \frac{\partial B}{\partial t} &= \omega B_{\max} \sin(kx - \omega t)
 \end{aligned}$$

Slide 13.38

place in $\Rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$

$$\Rightarrow kE_{\max} = \omega B_{\max}$$

$$\frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = c$$

$$\frac{E_{\max}}{B_{\max}} = \frac{E}{B} = c$$

That is, **at every instant**, the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light.

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})}} = 3.00 \times 10^8 \text{ m/s.}$$

This is a remarkable result. For this is precisely equal to the measured speed of light!

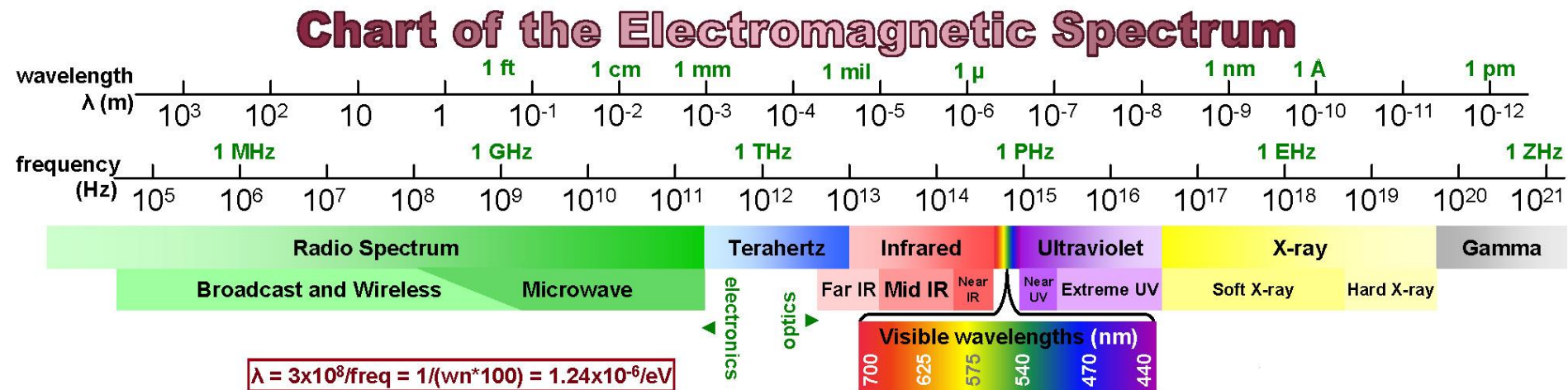
Light as an Electromagnetic Wave and the Electromagnetic Spectrum

$$c = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

The wavelengths of visible light were measured in the first decade of the nineteenth century, long before anyone imagined that light was an electromagnetic wave. The wavelengths were found to lie between $4.0 \times 10^{-7} \text{ m}$ and $7.5 \times 10^{-7} \text{ m}$, or 400 nm to 750 nm ($1 \text{ nm} = 10^{-9} \text{ m}$). The frequencies of visible light can be found

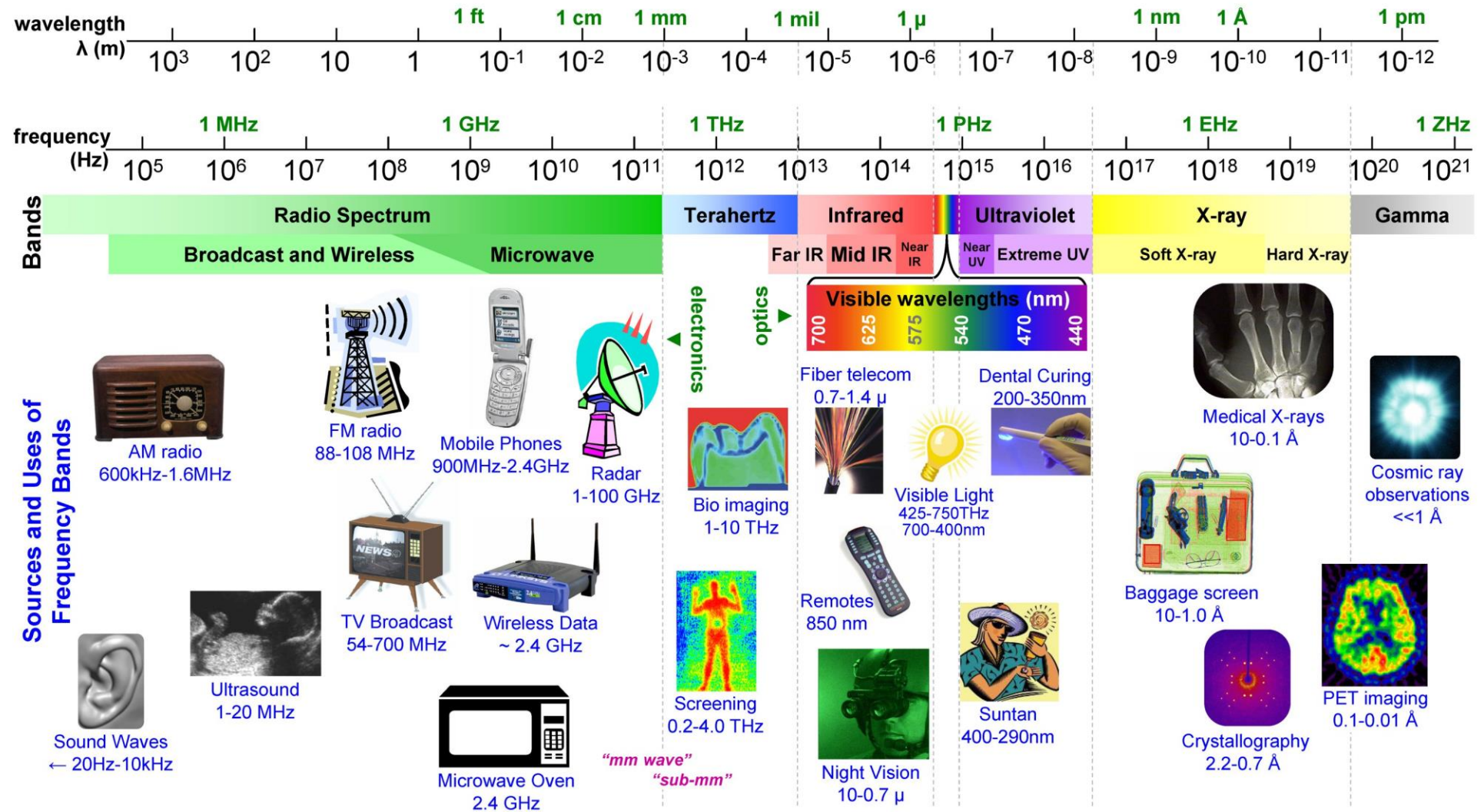
$$c = \lambda f$$

where f and λ are the frequency and wavelength, respectively, of the wave. Here, c is the speed of light, $3.00 \times 10^8 \text{ m/s}$; it gets the special symbol c because of its universality for all EM waves in free space.



- enormous range of wave lengths and frequencies
- spans more than 15 orders of magnitude

Applications of electromagnetic waves



$$\lambda = 3 \times 10^8 / \text{freq} = 1 / (\text{wn} \times 100) = 1.24 \times 10^{-6} / \text{eV}$$

Energy in EM Waves

Electromagnetic waves carry energy from one region of space to another. This energy is associated with the moving electric and magnetic fields. The energy density u_E (J/m³) stored in an electric field E is $u_E = \frac{1}{2} \epsilon_0 E^2$. The energy density stored in a magnetic field B is given by $u_B = \frac{1}{2} B^2 / \mu_0$. Thus, the total energy stored per unit volume in a region of space where there is an electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}. \quad (\text{total energy density associated to an EM wave})$$

In this equation, E and B represent the electric and magnetic field strengths of the wave at any instant in a small region of space. We can write u in terms of the E field alone, using $(B = E/c)$ and $(c = 1/\sqrt{\epsilon_0 \mu_0})$ to obtain

$$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{\epsilon_0 \mu_0 E^2}{\mu_0} = \epsilon_0 E^2.$$

Note here that the energy density associated with the B field equals that due to the E field, and each contributes half to the total energy. We can also write the energy density in terms of the B field only:

$$u = \epsilon_0 E^2 = \epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0},$$

or in one term containing both E and B ,

$$u = \epsilon_0 E^2 = \epsilon_0 E c B = \frac{\epsilon_0 E B}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}} E B.$$

The Poynting Vector

The rate of transfer of energy by an electromagnetic wave is described by a vector $\vec{S}$, called the Poynting vector, which is defined by the expression

$$\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The magnitude of the Poynting vector represents the rate at which energy passes through a unit surface area perpendicular to the direction of wave propagation in the unit time.

Therefore, the magnitude of $\vec{S}$ represents **power per unit area**.

The direction of the vector is along the direction of wave propagation.

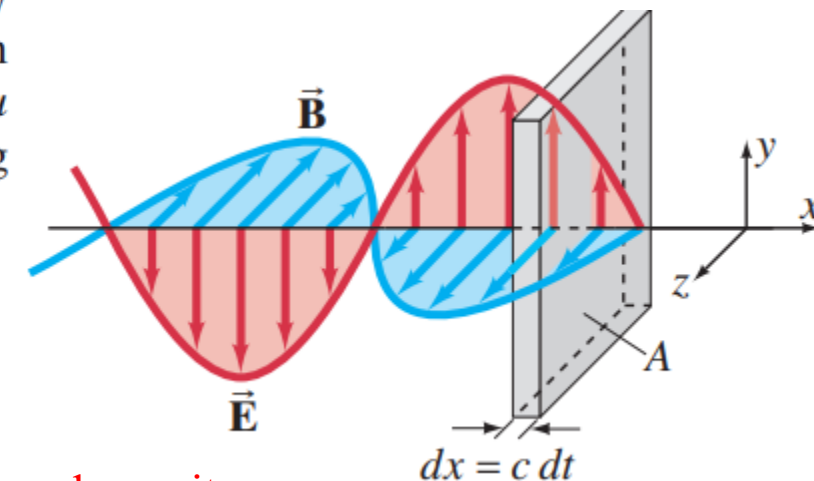
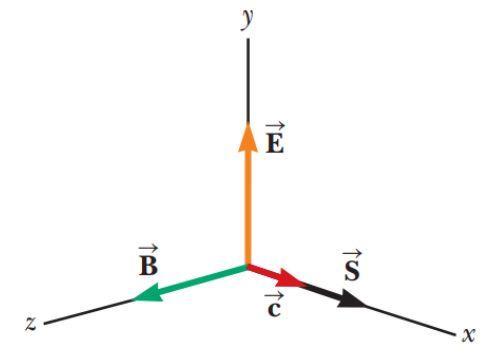
Let us imagine the wave is passing through an area A perpendicular to the x axis as shown in Fig. . In a short time dt , the wave moves to the right a distance $dx = c dt$ where c is the wave speed. The energy that passes through A in the time dt is the energy that occupies the volume $dV = A dx = Ac dt$. The energy density u is $u = \epsilon_0 E^2$ where E is the electric field in this volume at the given instant. So the total energy dU contained in this volume dV is the energy density u times the volume: $dU = u dV = (\epsilon_0 E^2)(Ac dt)$. Therefore the energy crossing the area A per time dt is

$$S = \frac{1}{A} \frac{dU}{dt} = \epsilon_0 c E^2.$$

$$E = cB \text{ and } c = 1/\sqrt{\epsilon_0 \mu_0}$$

$$S = \epsilon_0 c E^2 = \frac{c B^2}{\mu_0} = \frac{EB}{\mu_0} = cu$$

Instantaneous rate at which the energy is passing through a unit area



The Poynting Vector

We are often interested, for a sinusoidal plane electromagnetic wave, to the **time average of $\mathbf{S}$ over one or more cycles**, which is called the **wave intensity I** .

The **total instantaneous energy density u** is equal to the sum of the energy densities associated with the electric and magnetic fields:

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0} \quad (\text{see slide 13.45})$$

When this total instantaneous energy density is averaged over one or more cycles of an electromagnetic wave, we again obtain a factor of $\frac{1}{2}$. Hence, for any electromagnetic wave, the total average energy per unit volume is

$$u_{\text{avg}} = \epsilon_0 (E^2)_{\text{avg}} = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 = \frac{B_{\text{max}}^2}{2\mu_0}$$

$$I = S_{\text{avg}} = cu_{\text{avg}}$$

S_{avg} is also sometimes indicated as $\bar{S}$

In other words, the intensity of an electromagnetic wave equals the average energy density multiplied by the speed of light.

Momentum and Radiation Pressure

If ΔU is the energy absorbed by the object in a time Δt , there will be a net momentum Δp transferred to the object

[†]Very roughly, if we think of light as particles (and we do), the force that would be needed to bring such a particle moving at speed c to “rest” (i.e. absorption) is $F = \Delta p / \Delta t$. But F is also related to energy by $F = \Delta U / \Delta x$, so $\Delta p = F \Delta t = \Delta U / (\Delta x / \Delta t) = \Delta U / c$ where we identify $(\Delta x / \Delta t)$ with the speed of light c .

If electromagnetic waves carry energy, then we might expect them to also carry linear momentum.

When an electromagnetic wave encounters the surface of an object, a force will be exerted on the surface as a result of the momentum transfer just as when a moving object strikes a surface.

The force per unit area exerted by the waves is called radiation pressure.

If a beam of EM radiation (light, for example) is completely absorbed by an object, then the momentum transferred is

ΔU is the energy absorbed by the object in a time Δt

$$\Delta p = \frac{\Delta U}{c} \quad \left[\begin{array}{c} \text{radiation} \\ \text{fully} \\ \text{absorbed} \end{array} \right]$$

If instead, the radiation is fully reflected (suppose the object is a mirror), then the momentum transferred is twice as great, just as when a ball bounces elastically off a surface

$$\Delta p = \frac{2 \Delta U}{c} \quad \left[\begin{array}{c} \text{radiation} \\ \text{fully} \\ \text{reflected} \end{array} \right]$$

Momentum and Radiation Pressure

Here, S_{avg} is also sometimes indicated as $\bar{S}$

Using Newton's second law we can calculate the force and the pressure exerted by radiation on the object. The force F is given by

$$F = \frac{dp}{dt}, \quad \text{where } p \text{ is the momentum}$$

The average rate that energy is delivered to the object is related to the Poynting vector by

$$\frac{dU}{dt} = \bar{S}A, \quad (\text{see slide 13.46})$$

where A is the cross-sectional area of the object which intercepts the radiation.

The radiation pressure P (assuming full absorption $dp = dU/c$, see slide 13.48)

$$P = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{1}{Ac} \frac{dU}{dt} = \frac{\bar{S}}{c}. \quad \left[\begin{array}{c} \text{fully} \\ \text{absorbed} \end{array} \right]$$

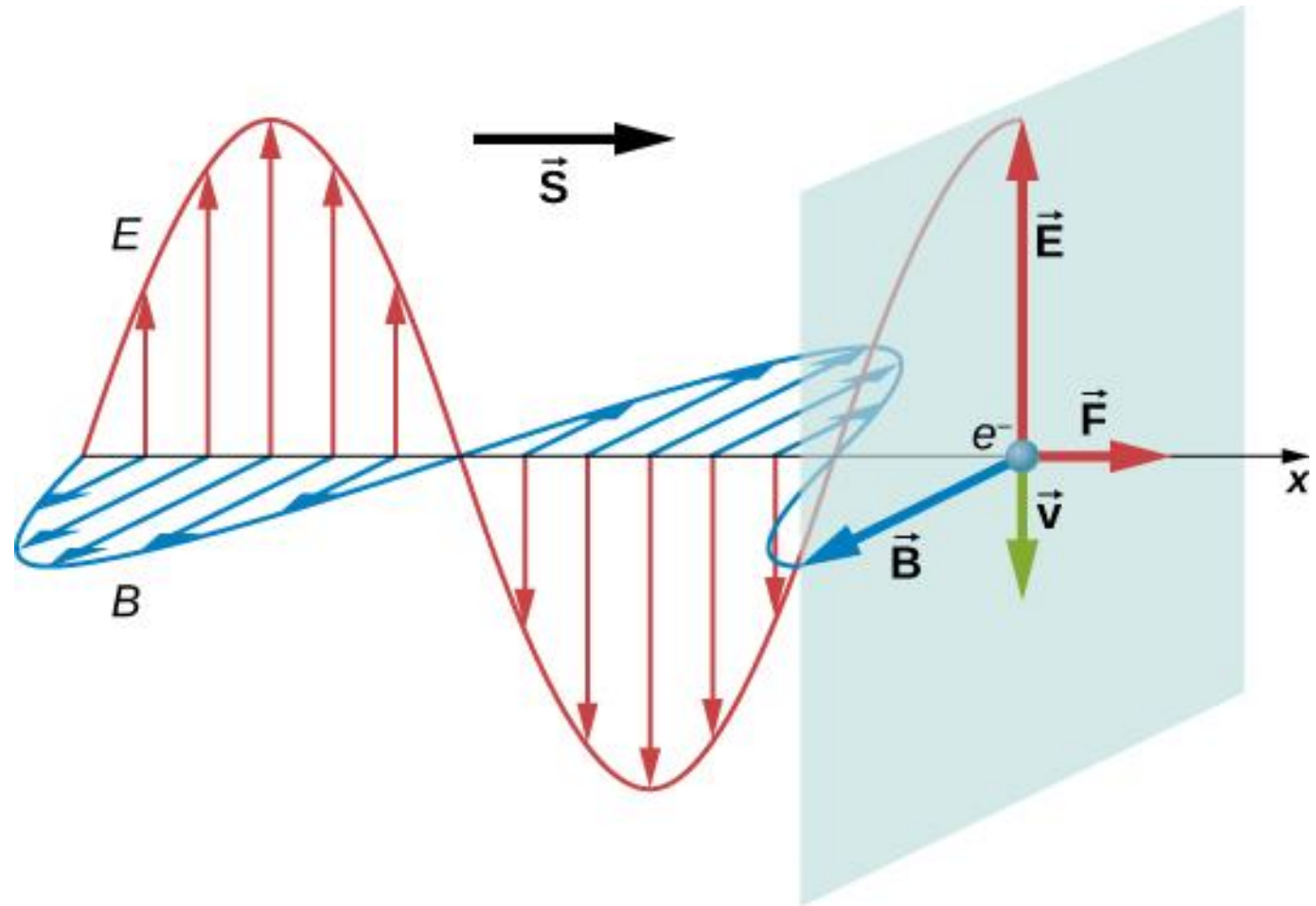
If the light is fully reflected, the pressure is twice as great

$$P = \frac{2\bar{S}}{c}. \quad \left[\begin{array}{c} \text{fully} \\ \text{reflected} \end{array} \right]$$

Radiation pressure: intuitive explanation for a metal

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Electric and magnetic fields of an electromagnetic wave can combine to produce a force in the direction of propagation, as illustrated for the special case of electrons whose motion is highly damped by the resistance of a metal.



By applying the right-hand rule, and accounting for the negative charge of the electron, we can see that the force on the electron from the magnetic field is in the direction of the positive x-axis, which is the direction of wave propagation.

Summary: Electromagnetic plane waves in vacuum

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{x} - \omega t + \varphi_0)$$

$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0 \cos(\mathbf{k} \cdot \mathbf{x} - \omega t + \varphi_0)$$

wave vector: $\mathbf{k}$

pulsation: $\omega = c|\mathbf{k}| = ck$

wavelength: $\lambda = \frac{2\pi}{|\mathbf{k}|}$

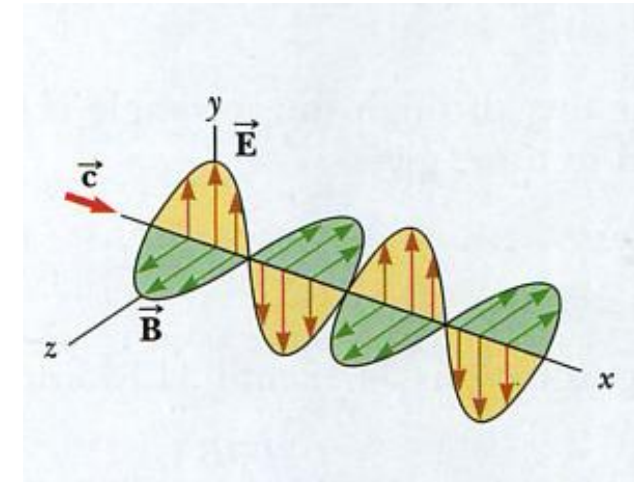
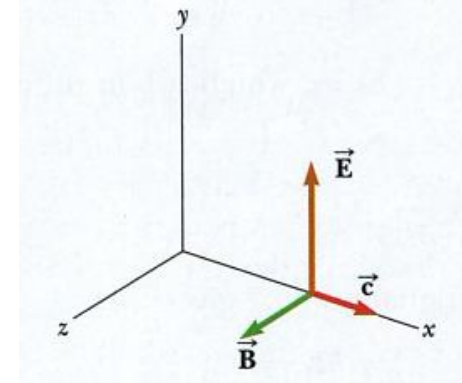
direction of propagation: $\hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|}$

Link between $\mathbf{E}_0$ et $\mathbf{B}_0$:

$$\mathbf{B}_0 = \frac{k}{\omega} (\hat{\mathbf{k}} \times \mathbf{E}_0) = \frac{1}{c} (\hat{\mathbf{k}} \times \mathbf{E}_0) \Rightarrow$$

$\mathbf{E}_0$ and $\mathbf{B}_0$ are:

- perpendicular to the direction of propagation" ("transverse wave") (i.e., $\mathbf{B} \perp \mathbf{k}, \mathbf{E} \perp \mathbf{k}$)
- perpendicular to each other (i.e., $\mathbf{B} \perp \mathbf{E}$)
- $|\mathbf{B}_0| = (1/c)|\mathbf{E}_0|$



Summary: Momentum of an EM wave in a vacuum

There is a momentum associated with the EM field

$$\mathbf{p} = \varepsilon_0 \mathbf{E} \times \mathbf{B} = \frac{\mathbf{S}}{c^2}$$

Density of momentum transported by the wave
[(mass x speed)/volume]

Instantaneous

Averages

Energy density:

$$u_{EM} = \frac{\varepsilon_0 |\mathbf{E}|^2}{2} + \frac{|\mathbf{B}|^2}{2\mu_0} \Rightarrow \text{for monochromatic plane wave: } \langle u_{EM} \rangle = \frac{1}{2} \varepsilon_0 E_0^2 = c \langle |\mathbf{p}| \rangle$$

Power through a unit area:

$$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \Rightarrow \text{for monochromatic plane wave: } \langle |\mathbf{S}| \rangle = \frac{1}{2} c \varepsilon_0 E_0^2 = c \langle u_{EM} \rangle = I_{avg}$$

Density of momentum:

$$\mathbf{p} = \varepsilon_0 (\mathbf{E} \times \mathbf{B}) = \frac{\mathbf{S}}{c^2} \Rightarrow \text{for monochromatic plane wave: } \langle |\mathbf{p}| \rangle = \frac{1}{2c} \varepsilon_0 E_0^2 = \frac{1}{c^2} \langle |\mathbf{S}| \rangle = \frac{I_{avg}}{c^2}$$

$$\text{with } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cong 3 \times 10^8 \text{ m/s}$$

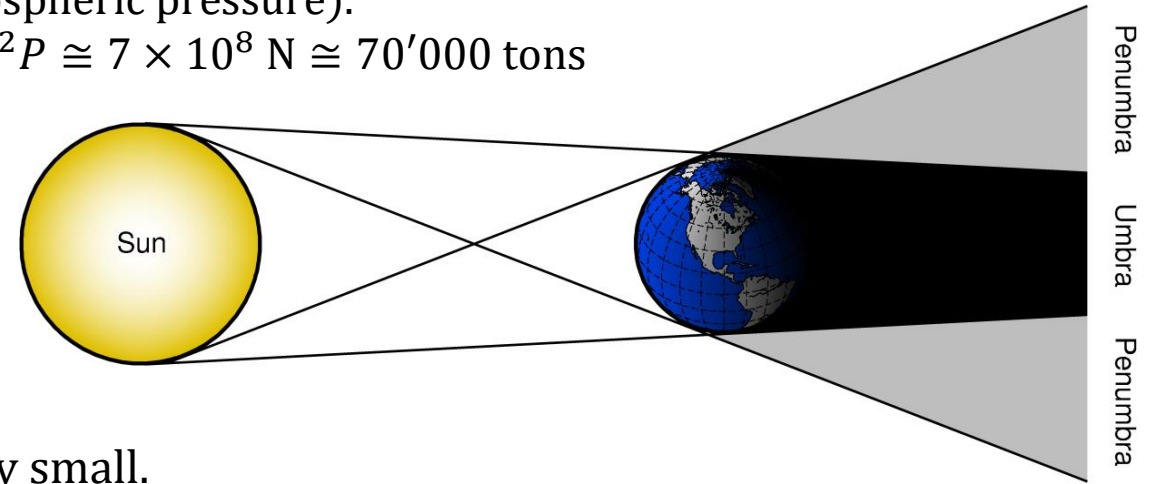
$$[\mathbf{S}] = \left[\frac{\text{W}}{\text{m}^2} \right]$$

$$[\mathbf{p}] = \left[\frac{\text{W}}{\text{m}^2} \frac{\text{s}^2}{\text{m}^2} \right] = \left[\text{kg} \frac{\text{m}}{\text{s}} \frac{1}{\text{m}^3} \right] = \left[\frac{\text{mass} \times \text{velocity}}{\text{volume}} \right]$$

Notes:

1) The radiation pressure of sunlight on a mirror on Earth is of the order of 10^{-11} bar ($P \cong 6 \times 10^{-6}$ Pa) (i.e., 10^{-11} lower than atmospheric pressure).

The total force of sunlight on the entire Earth is $F_{tot} = \pi R^2 P \cong 7 \times 10^8 \text{ N} \cong 70'000 \text{ tons}$

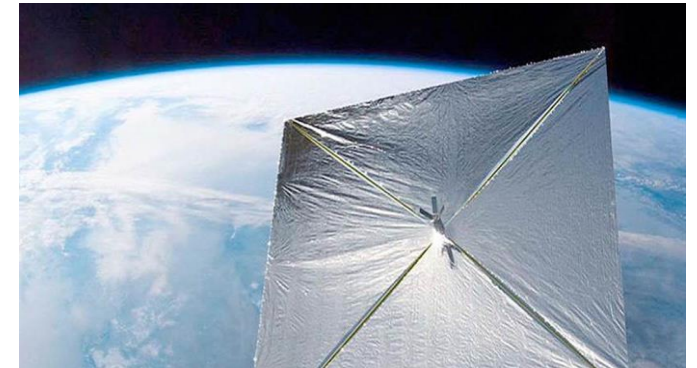


2) The forces generated by radiation pressure are usually small.

However, they play a crucial role in certain contexts, such as astrodynamics.

For example:

- if the effects of solar radiation pressure on the spacecraft of the Viking program had been ignored, the spacecraft would have missed the orbit of Mars by about 15,000 km.
- The Japan Aerospace Exploration Agency ("JAXA") has successfully deployed a 'solar sail' in space which has already successfully propelled its payload with the IKAROS project.



Some solved problems

EXAMPLE 2 Determining $\vec{E}$ and $\vec{B}$ in EM waves. Assume a 60.0-Hz EM wave is a sinusoidal wave propagating in the z direction with $\vec{E}$ pointing in the x direction, and $E_0 = 2.00 \text{ V/m}$. Write vector expressions for $\vec{E}$ and $\vec{B}$ as functions of position and time.

APPROACH We find λ from $\lambda f = v = c$. Then we use Fig. 9 and Eqs. 7 and 8 for the mathematical form of traveling electric and magnetic fields of an EM wave.

SOLUTION The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60.0 \text{ s}^{-1}} = 5.00 \times 10^6 \text{ m}.$$

From Eq. 8 we have

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{5.00 \times 10^6 \text{ m}} = 1.26 \times 10^{-6} \text{ m}^{-1}$$

$$\omega = 2\pi f = 2\pi(60.0 \text{ Hz}) = 377 \text{ rad/s}.$$

From Eq. 11 with $v = c$, we find that

$$B_0 = \frac{E_0}{c} = \frac{2.00 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 6.67 \times 10^{-9} \text{ T}.$$

The direction of propagation is that of $\vec{E} \times \vec{B}$, as in Fig. 9. With $\vec{E}$ pointing in the x direction, and the wave propagating in the z direction, $\vec{B}$ must point in the y direction. Using Eqs. 7 we find:

$$\vec{E} = \hat{i}(2.00 \text{ V/m}) \sin[(1.26 \times 10^{-6} \text{ m}^{-1})z - (377 \text{ rad/s})t]$$

$$\vec{B} = \hat{j}(6.67 \times 10^{-9} \text{ T}) \sin[(1.26 \times 10^{-6} \text{ m}^{-1})z - (377 \text{ rad/s})t]$$

EXAMPLE 3 Wavelengths of EM waves. Calculate the wavelength (λ) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave, and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

APPROACH All of these waves are electromagnetic waves, so their speed is $c = 3.00 \times 10^8$ m/s. We solve for λ in Eq. 14: $\lambda = c/f$.

SOLUTION (a) $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60 \text{ s}^{-1}} = 5.0 \times 10^6 \text{ m},$

or 5000 km. 60 Hz is the frequency of ac current in the United States, and, as we see here, one wavelength stretches all the way across the continental USA.

(b) $\lambda = \frac{3.00 \times 10^8 \text{ m/s}}{93.3 \times 10^6 \text{ s}^{-1}} = 3.22 \text{ m}.$

The length of an FM antenna is about half this ($\frac{1}{2}\lambda$), or $1\frac{1}{2}$ m.

(c) $\lambda = \frac{3.00 \times 10^8 \text{ m/s}}{4.74 \times 10^{14} \text{ s}^{-1}} = 6.33 \times 10^{-7} \text{ m} (= 633 \text{ nm}).$

EXAMPLE 4 ESTIMATE Cell phone antenna. The antenna of a cell phone is often $\frac{1}{4}$ wavelength long. A particular cell phone has an 8.5-cm-long straight rod for its antenna. Estimate the operating frequency of this phone.

APPROACH The basic equation relating wave speed, wavelength, and frequency is $c = \lambda f$; the wavelength λ equals four times the antenna's length.

SOLUTION The antenna is $\frac{1}{4}\lambda$ long, so $\lambda = 4(8.5 \text{ cm}) = 34 \text{ cm} = 0.34 \text{ m}$. Then $f = c/\lambda = (3.0 \times 10^8 \text{ m/s})/(0.34 \text{ m}) = 8.8 \times 10^8 \text{ Hz} = 880 \text{ MHz}$.

NOTE Radio antennas are not always straight conductors. The conductor may be a round loop to save space. See Fig. 21b.

EXAMPLE 5 ESTIMATE Phone call time lag. You make a telephone call from New York to a friend in London. Estimate how long it will take the electrical signal generated by your voice to reach London, assuming the signal is (a) carried on a telephone cable under the Atlantic Ocean, and (b) sent via satellite 36,000 km above the ocean. Would this cause a noticeable delay in either case?

APPROACH The signal is carried on a telephone wire or in the air via satellite. In either case it is an electromagnetic wave. Electronics as well as the wire or cable slow things down, but as a rough estimate we take the speed to be $c = 3.0 \times 10^8 \text{ m/s}$.

SOLUTION The distance from New York to London is about 5000 km.

(a) The time delay via the cable is $t = d/c \approx (5 \times 10^6 \text{ m})/(3.0 \times 10^8 \text{ m/s}) = 0.017 \text{ s}$.

(b) Via satellite the time would be longer because communications satellites, which are usually geosynchronous, move at a height of 36,000 km. The signal would have to go up to the satellite and back down, or about 72,000 km. The actual distance the signal would travel would be a little more than this as the signal would go up and down on a diagonal. Thus $t = d/c \approx 7.2 \times 10^7 \text{ m}/(3 \times 10^8 \text{ m/s}) = 0.24 \text{ s}$.

NOTE When the signal travels via the underwater cable, there is only a hint of a delay and conversations are fairly normal. When the signal is sent via satellite, the delay is noticeable. The length of time between the end of when you speak and your friend receives it and replies, and then you hear the reply, is about a half second beyond the normal time in a conversation. This is enough to be noticeable, and you have to adjust for it so you don't start talking again while your friend's reply is on the way back to you.

EXAMPLE 6 *E and B from the Sun.* Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about $1350 \text{ J/s} \cdot \text{m}^2 (= 1350 \text{ W/m}^2)$. Assume that this is a single EM wave, and calculate the maximum values of E and B .

APPROACH We solve Eq. 19a ($\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2$) for E_0 in terms of $\bar{S}$ using $\bar{S} = 1350 \text{ J/s} \cdot \text{m}^2$.

SOLUTION

$$E_0 = \sqrt{\frac{2\bar{S}}{\epsilon_0 c}} = \sqrt{\frac{2(1350 \text{ J/s} \cdot \text{m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}}$$

$$= 1.01 \times 10^3 \text{ V/m}.$$

From Eq. 11, $B = E/c$, so

$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 3.37 \times 10^{-6} \text{ T}.$$

NOTE Although B has a small numerical value compared to E (because of the way the different units for E and B are defined), B contributes the same energy to the wave as E does, as we saw earlier (Eqs. 15 and 16).

EXAMPLE 7 **ESTIMATE** **Solar pressure.** Radiation from the Sun that reaches the Earth's surface (after passing through the atmosphere) transports energy at a rate of about 1000 W/m^2 . Estimate the pressure and force exerted by the Sun on your outstretched hand.

APPROACH The radiation is partially reflected and partially absorbed, so let us estimate simply $P = \bar{S}/c$.

SOLUTION
$$P \approx \frac{\bar{S}}{c} = \frac{1000 \text{ W/m}^2}{3 \times 10^8 \text{ m/s}} \approx 3 \times 10^{-6} \text{ N/m}^2.$$

An estimate of the area of your outstretched hand might be about 10 cm by 20 cm, so $A = 0.02 \text{ m}^2$. Then the force is

$$F = PA \approx (3 \times 10^{-6} \text{ N/m}^2)(0.02 \text{ m}^2) \approx 6 \times 10^{-8} \text{ N}.$$

NOTE These numbers are tiny. The force of gravity on your hand, for comparison, is maybe a half pound, or with $m = 0.2 \text{ kg}$, $mg \approx (0.2 \text{ kg})(9.8 \text{ m/s}^2) \approx 2 \text{ N}$. The radiation pressure on your hand is imperceptible compared to gravity.

EXAMPLE 8 ESTIMATE

A solar sail. Proposals have been made to use the radiation pressure from the Sun to help propel spacecraft around the solar system. (a) About how much force would be applied on a $1\text{ km} \times 1\text{ km}$ highly reflective sail, and (b) by how much would this increase the speed of a 5000-kg spacecraft in one year? (c) If the spacecraft started from rest, about how far would it travel in a year?

APPROACH Pressure P is force per unit area, so $F = PA$. We use the estimate of Example 7, doubling it for a reflecting surface $P = 2\bar{S}/c$. We find the acceleration from Newton's second law, and assume it is constant, and then find the speed from $v = v_0 + at$. The distance traveled is given by $x = \frac{1}{2}at^2$.

SOLUTION (a) Doubling the result of Example 7, the solar pressure is $2\bar{S}/c = 6 \times 10^{-6}\text{ N/m}^2$. Then the force is $F \approx PA = (6 \times 10^{-6}\text{ N/m}^2)(10^6\text{ m}^2) \approx 6\text{ N}$. (b) The acceleration is $a \approx F/m \approx (6\text{ N})/(5000\text{ kg}) \approx 1.2 \times 10^{-3}\text{ m/s}^2$. The speed increase is $v - v_0 = at = (1.2 \times 10^{-3}\text{ m/s}^2)(365\text{ days})(24\text{ hr/day})(3600\text{ s/hr}) \approx 4 \times 10^4\text{ m/s}$ ($\approx 150,000\text{ km/h!}$). (c) Starting from rest, this acceleration would result in a distance of about $\frac{1}{2}at^2 \approx 6 \times 10^{11}\text{ m}$ in a year, about four times the Sun-Earth distance. The starting point should be far from the Earth so the Earth's gravitational force is small compared to 6 N.

NOTE A large sail providing a small force over a long time can result in a lot of motion.

✓ Light Bulb Fields

A light bulb emits 5.00 W of power as visible light. What are the average electric and magnetic fields from the light at a distance of 3.0 m?

Strategy

Assume the bulb's power output $\mathbf{P}$ is distributed uniformly over a sphere of radius 3.0 m to calculate the intensity, and from it, the electric field.

Solution

The power radiated as visible light is then

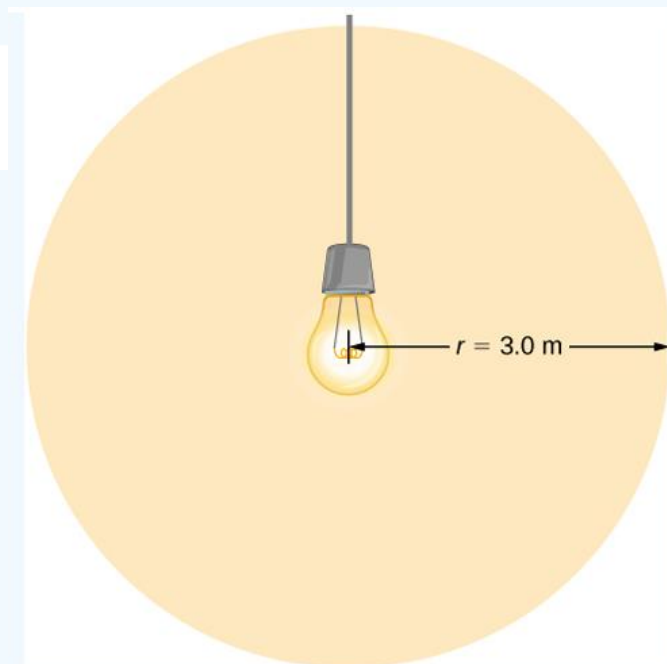
$$I = \frac{P}{4\pi r^2} = \frac{c\epsilon_0 E_0^2}{2},$$

$$E_0 = \sqrt{2 \frac{P}{4\pi r^2 c\epsilon_0}} = \sqrt{2 \frac{5.00 \text{ W}}{4\pi (3.0 \text{ m})^2 (3.00 \times 10^8 \text{ m/s}) (8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)}} = 5.77 \text{ N/C},$$

$$B_0 = E_0/c = 1.92 \times 10^{-8} \text{ T}.$$

Significance

The intensity $\mathbf{I}$ falls off as the distance squared if the radiation is dispersed uniformly in all directions.



✓ Radio Range

A 60-kW radio transmitter on Earth sends its signal to a satellite 100 km away (Figure 16.4.3). At what distance in the same direction would the signal have the same maximum field strength if the transmitter's output power were increased to 90 kW?

The area over which the power in a particular direction is dispersed increases as distance squared, as illustrated in Figure 16.4.3. Change the power output P by a factor of (90 kW/60 kW) and change the area by the same factor to keep $I = \frac{P}{A} = \frac{c\epsilon_0 E_0^2}{2}$ the same. Then use the proportion of area A in the diagram to distance squared to find the distance that produces the calculated change in area.

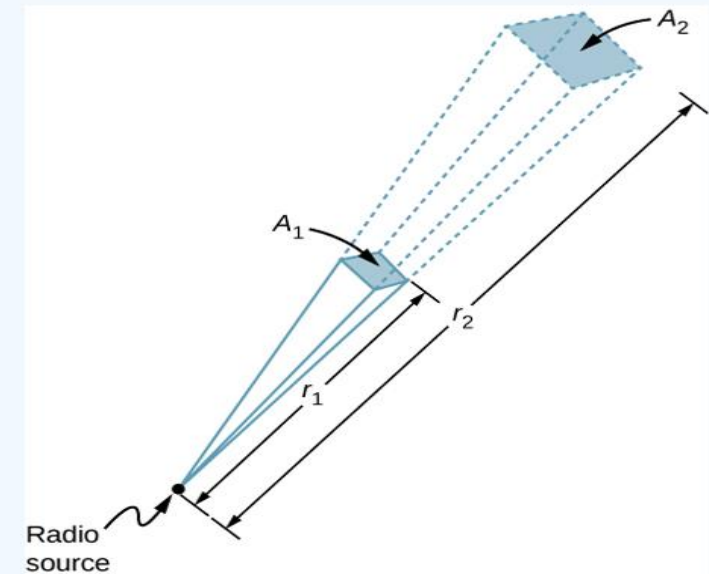
Solution

Using the proportionality of the areas to the squares of the distances, and solving, we obtain from the diagram

$$\begin{aligned}\frac{r_2^2}{r_1^2} &= \frac{A_2}{A_1} = \frac{90 \text{ W}}{60 \text{ W}}, \\ r_2 &= \sqrt{\frac{90}{60}}(100 \text{ km}) \\ &= 122 \text{ km}.\end{aligned}$$

Significance

The range of a radio signal is the maximum distance between the transmitter and receiver that allows for normal operation. In the absence of complications such as reflections from obstacles, the intensity follows an inverse square law, and doubling the range would require multiplying the power by four.



Example 34.2 Fields on the Page

Estimate the maximum magnitudes of the electric and magnetic fields of the light that is incident on this page because of the visible light coming from your desk lamp. Treat the bulb as a point source of electromagnetic radiation that is 5% efficient at transforming energy coming in by electrical transmission to energy leaving by visible light.

Solution Recall from Equation 17.7 that the wave intensity I at a distance r from a point source is $I = \mathcal{P}_{\text{av}}/4\pi r^2$, where $\mathcal{P}_{\text{av}}$ is the average power output of the source and $4\pi r^2$ is the area of a sphere of radius r centered on the source. Because the intensity of an electromagnetic wave is also given by Equation 34.21, we have

$$I = \frac{\mathcal{P}_{\text{av}}}{4\pi r^2} = \frac{E_{\text{max}}^2}{2\mu_0 c}$$

We must now make some assumptions about numbers to enter in this equation. If we have a 60-W lightbulb, its output at 5% efficiency is approximately 3.0 W by visible light. (The remaining energy transfers out of the bulb by conduction and invisible radiation.) A reasonable distance from the bulb to the page might be 0.30 m. Thus, we have

$$\begin{aligned} E_{\text{max}} &= \sqrt{\frac{\mu_0 c \mathcal{P}_{\text{av}}}{2\pi r^2}} \\ &= \sqrt{\frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(3.00 \times 10^8 \text{ m/s})(3.0 \text{ W})}{2\pi(0.30 \text{ m})^2}} \\ &= 45 \text{ V/m} \end{aligned}$$

From Equation 34.14,

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{45 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.5 \times 10^{-7} \text{ T}$$

This value is two orders of magnitude smaller than the Earth's magnetic field, which, unlike the magnetic field in the light wave from your desk lamp, is not oscillating.

Example 34.5 Solar Energy

As noted in the preceding example, the Sun delivers about 10^3 W/m^2 of energy to the Earth's surface via electromagnetic radiation.

(A) Calculate the total power that is incident on a roof of dimensions $8.00 \text{ m} \times 20.0 \text{ m}$.

Solution We assume that the average magnitude of the Poynting vector for solar radiation at the surface of the Earth is $S_{\text{av}} = 1\,000 \text{ W/m}^2$; this represents the power per unit area, or the light intensity. Assuming that the radiation is incident normal to the roof, we obtain

$$\begin{aligned}\mathcal{P}_{\text{av}} &= S_{\text{av}} A = (1\,000 \text{ W/m}^2)(8.00 \times 20.0 \text{ m}^2) \\ &= 1.60 \times 10^5 \text{ W}\end{aligned}$$

What If? Suppose the energy striking the roof could be captured and used to operate electrical devices in the house. Could the home operate completely from this energy?

Answer The power in part (A) is large compared to the power requirements of a typical home. If this power were maintained for 24 hours per day and the energy could be absorbed and made available to electrical devices, it would provide more than enough energy for the average home. However, solar energy is not easily harnessed, and the prospects for large-scale conversion are not as bright as may appear from this calculation. For example, the efficiency of conversion from solar energy is typically 10% for

(B) Determine the radiation pressure and the radiation force exerted on the roof, assuming that the roof covering is a perfect absorber.

Solution Using Equation 34.25 with $S_{\text{av}} = 1\,000 \text{ W/m}^2$, we find that the radiation pressure is

$$P_{\text{av}} = \frac{S_{\text{av}}}{c} = \frac{1\,000 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ N/m}^2$$

Because pressure equals force per unit area, this corresponds to a radiation force of

$$\begin{aligned}F &= P_{\text{av}} A = (3.33 \times 10^{-6} \text{ N/m}^2)(160 \text{ m}^2) \\ &= 5.33 \times 10^{-4} \text{ N}\end{aligned}$$

photovoltaic cells, reducing the available power in part (A) by an order of magnitude. Other considerations reduce the power even further. Depending on location, the radiation will most likely not be incident normal to the roof and, even if it is (in locations near the Equator), this situation exists for only a short time near the middle of the day. No energy is available for about half of each day during the nighttime hours. Furthermore, cloudy days reduce the available energy. Finally, while energy is arriving at a large rate during the middle of the day, some of it must be stored for later use, requiring batteries or other storage devices. The result of these considerations is that complete solar operation of homes is not presently cost-effective for most homes.